ELECTROMAGNETIC WAVES IN MATTER - NORMAL REFLECTION AND TRANSMISSION

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To see how electromagnetic waves propagate in matter, we can start with Maxwell's equations in matter:

$$\nabla\cdot\mathbf{D} = \rho_f \tag{1}$$
$$\nabla\cdot\mathbf{B} = 0 \tag{2}$$
$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{3}$$
$$\nabla\times\mathbf{H} = \mathbf{J}_f+\frac{\partial\mathbf{D}}{\partial t} \tag{4}$$

If the medium is linear, homogeneous and contains no free charge or current, these equations reduce to

$$\nabla\cdot\mathbf{E} = 0 \tag{5}$$
$$\nabla\cdot\mathbf{B} = 0 \tag{6}$$
$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{7}$$
$$\nabla\times\mathbf{B} = \mu\epsilon\frac{\partial\mathbf{E}}{\partial t} \tag{8}$$

These equations are identical to those for a vacuum, except that the permeability ϵ_0 and permittivity μ_0 of free space have been replaced by their corresponding values ϵ and μ in the medium. Thus electromagnetic waves propagate the same way in a medium, so we can write them as

$$\tilde{\mathbf{E}} = \tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\hat{\mathbf{n}} \tag{9}$$
$$\tilde{\mathbf{B}} = \tilde{B}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\hat{\mathbf{k}}\times\hat{\mathbf{n}} \tag{10}$$
$$= \frac{1}{v}\hat{\mathbf{k}}\times\tilde{\mathbf{E}} \tag{11}$$

where

$$v=\frac{1}{\sqrt{\mu\epsilon}} \tag{12}$$

is the speed of the wave in the medium. Since the permittivities and permeabilities of materials are almost always greater than those for free space, the speed $v<c$ in almost all cases, so light travels more slowly through a medium than through a vacuum. The ratio of the speeds is the *index of refraction*:

$$n\equiv\frac{c}{v}=\sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}} \tag{13}$$

We can apply the boundary conditions at the interface between two media to find out how light behaves when passing from one medium into another.

$$\epsilon_1 E_1^{\perp}-\epsilon_2 E_2^{\perp}=0 \tag{14}$$

$$B_1^{\perp}-B_2^{\perp}=0 \tag{15}$$

$$\mathbf{E}_1^{\|}-\mathbf{E}_2^{\|}=0 \tag{16}$$

$$\frac{1}{\mu_1}\mathbf{B}_1^{\|}-\frac{1}{\mu_2}\mathbf{B}_2^{\|}=0 \tag{17}$$

We'll start with an incident wave travelling in the $+z$ direction (so $\hat{\mathbf{k}}=\hat{\mathbf{z}}$) and polarized in the x direction (so $\hat{\mathbf{n}}=\hat{\mathbf{x}}$) and suppose the boundary is the xy plane, with medium 1 on the left ($z<0$) and medium 2 on the right ($z>0$). Then the incident wave is

$$\tilde{\mathbf{E}}_I=\tilde{E}_{0_I}e^{i(k_1z-\omega t)}\hat{\mathbf{x}} \tag{18}$$

$$\tilde{\mathbf{B}}_I=\tilde{B}_{0_I}e^{i(k_1z-\omega t)}\hat{\mathbf{z}}\times\hat{\mathbf{x}} \tag{19}$$

$$=\frac{1}{v_1}\tilde{E}_{0_I}e^{i(k_1z-\omega t)}\hat{\mathbf{y}} \tag{20}$$

We'll have a reflected wave ($\hat{\mathbf{k}}=-\hat{\mathbf{z}}$) and a transmitted wave ($\hat{\mathbf{k}}=\hat{\mathbf{z}}$), so

$$\tilde{\mathbf{E}}_R=\tilde{E}_{0_R}e^{i(-k_1z-\omega t)}\left(\cos\theta_R\hat{\mathbf{x}}+\sin\theta_R\hat{\mathbf{y}}\right) \tag{21}$$

$$\tilde{\mathbf{B}}_R=-\tilde{B}_{0_R}e^{i(-k_1z-\omega t)}\hat{\mathbf{z}}\times\left(\cos\theta_R\hat{\mathbf{x}}+\sin\theta_R\hat{\mathbf{y}}\right) \tag{22}$$

$$=\frac{1}{v_1}\tilde{E}_{0_R}e^{i(-k_1z-\omega t)}\left(\sin\theta_R\hat{\mathbf{x}}-\cos\theta_R\hat{\mathbf{y}}\right) \tag{23}$$

$$\tilde{\mathbf{E}}_T = \tilde{E}_{0_T} e^{i(k_2 z - \omega t)} (\cos\theta_T \hat{\mathbf{x}} + \sin\theta_T \hat{\mathbf{y}}) \tag{24}$$

$$\tilde{\mathbf{B}}_T = \tilde{B}_{0_T} e^{i(k_2 z - \omega t)} \hat{\mathbf{z}} \times (\cos\theta_T \hat{\mathbf{x}} + \sin\theta_T \hat{\mathbf{y}}) \tag{25}$$

$$= \frac{1}{v_2} \tilde{E}_{0_T} e^{i(k_2 z - \omega t)} (-\sin\theta_T \hat{\mathbf{x}} + \cos\theta_T \hat{\mathbf{y}}) \tag{26}$$

where θ_R and θ_T are the angles of polarization for the reflected and transmitted waves.

Since there are no components of the fields perpendicular to the boundary at $z = 0$, 14 and 15 tell us nothing. Applying 16 to the x and y components, we have

$$\tilde{E}_{0_I} \hat{\mathbf{x}} + \tilde{E}_{0_R} (\cos\theta_R \hat{\mathbf{x}} + \sin\theta_R \hat{\mathbf{y}}) = \tilde{E}_{0_T} (\cos\theta_T \hat{\mathbf{x}} + \sin\theta_T \hat{\mathbf{y}}) \tag{27}$$

$$\tilde{E}_{0_I} + \tilde{E}_{0_R} \cos\theta_R = \tilde{E}_{0_T} \cos\theta_T \tag{28}$$

$$\tilde{E}_{0_R} \sin\theta_R = \tilde{E}_{0_T} \sin\theta_T \tag{29}$$

From 17 we have

$$\frac{1}{\mu_1 v_1} \tilde{E}_{0_R} \sin\theta_R = -\frac{1}{\mu_2 v_2} \tilde{E}_{0_T} \sin\theta_T \tag{30}$$

$$\frac{1}{\mu_1 v_1} \left(\tilde{E}_{0_I} - \tilde{E}_{0_R} \cos\theta_R \right) = \frac{1}{\mu_2 v_2} \tilde{E}_{0_T} \cos\theta_T \tag{31}$$

Substituting from 29 into 30 we have

$$\frac{1}{\mu_1 v_1} \tilde{E}_{0_T} \sin\theta_T = -\frac{1}{\mu_2 v_2} \tilde{E}_{0_T} \sin\theta_T \tag{32}$$

The only way this equation can be satisfied is if $\sin\theta_T = 0$ so from 29 we conclude that $\sin\theta_R = 0$ as well. Thus θ_T and θ_R are either 0 or π, so the cosines are ± 1. Thus the plane of polarization is the same for the reflected and transmitted waves as for the incident wave, and the reflected and transmitted waves are either in phase ($\theta = 0$) or half a cycle out of phase ($\theta = \pi$) with the incident wave..

If we define

$$\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} \tag{33}$$

we can write 31 as

$$\tilde{E}_{0_I} - \tilde{E}_{0_R} \cos\theta_R = \beta \tilde{E}_{0_T} \cos\theta_T \tag{34}$$

Adding this to 28 we get

$$\tilde{E}_{0_I} = \frac{1+\beta}{2}\tilde{E}_{0_T}\cos\theta_T \tag{35}$$

$$\tilde{E}_{0_T} = \pm\frac{2}{1+\beta}\tilde{E}_{0_I} \tag{36}$$

Subtracting 34 from 28 we get

$$\tilde{E}_{0_R} = \pm\frac{1-\beta}{2}\tilde{E}_{0_T} = \pm\frac{1-\beta}{1+\beta}\tilde{E}_{0_I} \tag{37}$$

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