FARADAY'S LAW

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The relation between emf and change of magnetic flux turns out to be a special case of a more general law discovered by Michael Faraday and called *Faraday's law*. It turns out that any change of magnetic flux through a loop, whatever the cause, results in an emf being generated around the loop. This can be caused by moving the loop relative to a fixed magnet, moving the magnet relative to a fixed loop, or keeping both loop and magnet fixed and varying the field strength.

Actually, it may seem surprising that the first two cases are treated separately; surely the relative motion of loop and magnet means they are both the same? Special relativity would, of course, confirm this, and it was partly this aspect of electromagnetism that inspired Einstein to think about relative motion. However, in the 19th century, motion was always considered relative to a fixed reference frame so the two cases were quite different. In particular, if the loop is considered fixed and the magnet moves, then the charges in the loop are at rest, so should not feel any magnetic force from the Lorentz force law. The fact that an emf *is* generated in this case as well led Faraday to postulate that a changing magnetic field produces an electric field. Faraday's law states that the emf generated in a loop is minus the rate of change of magnetic flux through the loop. That is

$$\boxed{\mathcal{E}=\oint\mathbf{E}\cdot d\boldsymbol{\ell}=-\frac{d\Phi}{dt}} \tag{1}$$

Since $\Phi=\int\mathbf{B}\cdot d\mathbf{a}$ then if the area enclosed by the loop stays the same, we have

$$\oint\mathbf{E}\cdot d\boldsymbol{\ell}=-\frac{d\Phi}{dt}=-\int\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{a} \tag{2}$$

From Stokes's theorem, we have

$$\oint\mathbf{E}\cdot d\boldsymbol{\ell}=\int(\nabla\times\mathbf{E})\cdot d\mathbf{a}=-\int\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{a} \tag{3}$$

so in differential form, we have

$$\boxed{\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}} \tag{4}$$

This is the generalization of the electrostatic condition $\nabla\times\mathbf{E}=0$ which applied in the absence of magnetic fields and is Faraday's law in differential form.

Faraday's law

Example 1. We have a solenoid with n turns per unit length in which the current in the wire wrapped around it varies with time according to $I(t)=I_0\cos\omega t$, so that the magnetic field inside the solenoid is

$$\mathbf{B}(t)=\hat{\mathbf{z}}B_0\cos\omega t \tag{5}$$

where

$$B_0=n\mu_0 I_0 \tag{6}$$

A circular loop of wire with resistance R and radius $a/2$ is placed inside the solenoid so that its axis is coincident with the solenoid's axis. The area of the loop is then $A=\frac{\pi a^2}{4}$ so the flux through the loop is

$$\Phi(t)=B\times A=\frac{\pi a^2 B_0}{4}\cos\omega t \tag{7}$$

so the emf generated in the loop is

$$\mathcal{E}=-\frac{d\Phi}{dt}=\frac{\pi a^2 B_0\omega}{4}\sin\omega t \tag{8}$$

and the current is

$$I=\frac{\mathcal{E}}{R}=\frac{\pi a^2 B_0\omega}{4R}\sin\omega t \tag{9}$$

To get the direction of the current, we need to define the directions of the vectors in 2. The test loop inside the solenoid is oriented perpendicular to the z axis, so we can take the area element $d\boldsymbol{a}$ to be along $\hat{\boldsymbol{z}}$. By the right-hand rule, the line integral in 2 is then taken in a counterclockwise direction around the loop when viewed from above, that is, when we're looking down the z axis in a direction opposite to $\hat{\boldsymbol{z}}$. Since $\boldsymbol{B}$ is parallel to $\hat{\boldsymbol{z}}$, then if the field is increasing (which happens for times when $\cos\omega t>0$), then $\boldsymbol{B}$ is parallel to $d\boldsymbol{a}$ and the term on the RHS of 2 is negative. This in turn means that the line intergral $\oint\mathbf{E}\cdot d\boldsymbol{\ell}$ must be negative, so the induced electric field and thus the current is clockwise when viewed down the z axis. Looked at another way, the minus sign in Faraday's law 4 says that the induced emf opposes the change in the magnetic field.

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