GYROMAGNETIC RATIO

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The *gyromagnetic ratio* γ is the ratio of an object's magnetic dipole moment to its angular momentum. The numerical value of γ depends on the units being used (that is, it's not a dimensionless quantity).

For a circular wire loop of radius r carrying charge Q and having mass M we can calculate γ as follows. Suppose the loop is rotating with angular speed ω. Then the current in the loop is $I=\frac{Q}{2\pi r}r\omega=\frac{Q}{2\pi}\omega$. The magnetic dipole moment is therefore

$$m=Ia=\frac{Q}{2\pi}\omega\pi r^{2}=\frac{Q}{2}\omega r^{2} \tag{1}$$

The angular momentum is $L=mr^{2}\omega$, so the gyromagnetic ratio of a circular wire loop is

$$\gamma=\frac{m}{L}=\frac{Q}{2M} \tag{2}$$

The ratio is independent of r and ω, which might seem a bit bizarre since we can calculate γ even when the loop isn't spinning, and therefore there is no current or angular momentum. In that case, γ should be interpreted as a limit, since otherwise it would involve dividing zero by zero.

Since the ratio is independent of r, it applies to any circular loop so we can apply this to any solid of revolution, such as a sphere or cylinder. All such objects have the same formula for γ.

We can attempt to find γ for an elementary particle such as an electron, although the resulting answer isn't correct (as you might expect when applying classical physics to quantum objects). The electron has spin $\frac{1}{2}$ which means its angular momentum is $\frac{\hbar}{2}$. We then get from 2

$$\gamma_{e}=\frac{2m_{e}}{\hbar}=-\frac{e}{2M_{e}} \tag{3}$$

where m_e is the dipole moment of the electron, M_e is its mass and e is the elementary charge. Plugging in the numbers we get (in SI units):

$$\gamma_e = -\frac{1.602 \times 10^{-19}}{2 \times 9.11 \times 10^{-31}} = -8.792 \times 10^{10} \text{C} \cdot \text{kg}^{-1} \tag{4}$$

From this we can get the magnetic dipole moment of the electron

$$m_e = \frac{\hbar}{2}\gamma_e = -4.636 \times 10^{-24} \text{Amp} \cdot \text{m}^2 \tag{5}$$

Experimentally, the value is $m_e = -9.285 \times 10^{-24}\text{Amp} \cdot \text{m}^2$ which is almost exactly twice the calculated value. When the moment is calculated using quantum electrodynamics, the agreement is pretty much exact.