

MAGNETIC FIELD OF A SOLENOID

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A solenoid is a helical coil of wire wound round an insulating cylinder. We can find the magnetic field due to a solenoid carrying a steady current I as follows. First, we work out the field due to a single circular loop of radius a as measured at a point P on the axis (which we'll take to be the z axis) of the loop. Fig. 1 shows a side view of the solenoid.

In the figure, we'll take the circular loop to be the first loop in the solenoid (closest to P), whose radius subtends an angle θ_1 at P . The distance from P along the axis to the centre of the circle is z . (We'll refer to the other parts of the diagram later.)

By symmetry, the components of the field in the x and y directions cancel, so we need calculate only the z component. The Biot-Savart law in this case is

$$\mathbf{B}(\mathbf{r}) = \hat{\mathbf{z}} \frac{I\mu_0}{4\pi} \int \frac{d\mathbf{l} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \quad (1)$$

The line element $d\mathbf{l}$ is tangent to the circle, and the vector $\mathbf{r} - \mathbf{r}'$ is always perpendicular to it. Look at the point C_1 on the loop where the tail of the θ_1

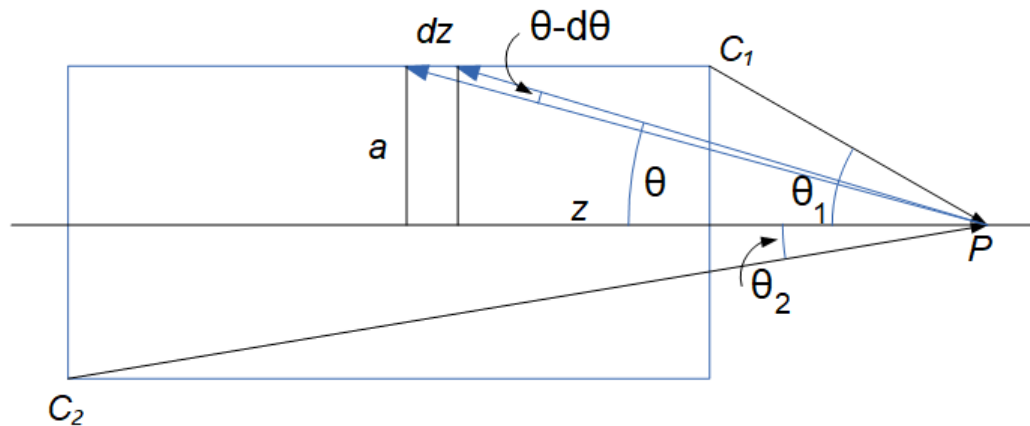


FIGURE 1. Side view of a solenoid.

vector meets the current loop, and assume the current is coming out of the page at this point. Then applying the right-hand rule, we see that the field vector lies in the plane of the figure, and points diagonally to the upper right, making an angle of $\frac{\pi}{2} - \theta_1$ with the z axis. To project out the z component of the field, we multiply by $\cos\left(\frac{\pi}{2} - \theta_1\right) = \sin\theta_1 = \frac{a}{|\mathbf{r} - \mathbf{r}'|}$. This angle is a constant, as is $|\mathbf{r} - \mathbf{r}'|$, and the integration extends round the circumference of the circle, so we get

$$B_z = \frac{I\mu_0}{4\pi} \frac{a}{|\mathbf{r} - \mathbf{r}'|} \frac{2\pi a}{|\mathbf{r} - \mathbf{r}'|^2} \quad (2)$$

$$= \frac{I\mu_0}{2a} \sin^3 \theta_1 \quad (3)$$

Now suppose the solenoid has n turns per unit length, and that this number is large enough that we can approximate the solenoid by a circular surface current density of In per unit length, and thus use integration to determine the overall field. To do the integral, we need to work out the relation between a change in θ and a change in z , as shown in the diagram. An increase of dz means a decrease in angle of $-d\theta$ as shown. To work out the relation:

$$\sin\theta = \frac{a}{\sqrt{a^2 + z^2}} \quad (4)$$

$$\sin(\theta - d\theta) = \frac{a}{\sqrt{a^2 + (z + dz)^2}} \quad (5)$$

We can expand the second equation in a Taylor series and retain only first terms in the differentials, and we get

$$\sin\theta - \cos\theta d\theta = \frac{a}{\sqrt{a^2 + z^2}} - \frac{az}{(a^2 + z^2)^{3/2}} dz \quad (6)$$

Replacing the terms on the right by trig functions, using

$$\cos\theta = \frac{z}{\sqrt{a^2 + z^2}} \quad (7)$$

we have

$$\frac{a}{\sqrt{a^2 + z^2}} - \frac{az}{(a^2 + z^2)^{3/2}} dz = \sin\theta - \frac{1}{a} \cos\theta \sin^2\theta dz \quad (8)$$

Thus we get

$$dz = \frac{a}{\sin^2\theta} d\theta \quad (9)$$

The current in an infinitesimal slice of the solenoid is therefore

$$Indz = \frac{Ina}{\sin^2 \theta} d\theta \quad (10)$$

We can now find the total field by doing the integral

$$B_z = \int_{\theta_2}^{\theta_1} \frac{\mu_0}{4\pi a} \sin^3 \theta \frac{Ina}{\sin^2 \theta} d\theta \quad (11)$$

$$= \frac{n\mu_0 I}{2} (\cos \theta_2 - \cos \theta_1) \quad (12)$$

This might look like it's independent of the radius a , but this dependence is included in the angles.

For an infinite solenoid, $\theta_1 \rightarrow \pi$ and $\theta_2 \rightarrow 0$ and we get

$$B_\infty = n\mu_0 I \quad (13)$$

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