MAGNETIC FLUX

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When we introduced motional emf, we saw that for a simple rectangular loop, the emf generated by pulling the loop through a magnetic field with a speed v, assuming that one side (of length ℓ) was inside the field region and moved perpendicular to its length, was

$$\mathcal{E}=\ell Bv \tag{1}$$

If the distance from the edge of the rectangle generating the emf to the edge of the field region is x, then as we pull the loop out of the field, the distance x inside the field region decreases with speed v. That is, $dx/dt=-v$. Since ℓ remains constant, the area of the loop inside the field is $A=\ell x$ and its rate of change is $dA/dt=\ell\frac{dx}{dt}=-\ell v$. We can therefore write the emf as

$$\mathcal{E}=-B\frac{dA}{dt} \tag{2}$$

At this point, we can define the *magnetic flux* Φ as the integral of the field over the area enclosed by the loop:

$$\Phi\equiv\int\mathbf{B}\cdot d\mathbf{a} \tag{3}$$

In this particular case, with $\mathbf{B}$ constant, we have

$$\mathcal{E}=-\frac{d\Phi}{dt} \tag{4}$$

In fact, this is a general relation that applies for any shape of loop and any magnetic field, even one that varies over space. The proof is given in Griffiths, so we won't repeat it here, as it doesn't really add anything to our understanding of the physics.

Note that we didn't specify the surface of integration in 3. In fact, the integral is independent of the surface as we can see by the following argument. Suppose we consider a closed surface (such as a sphere, but any closed surface will do). Then by Gauss's theorem

$$\int \mathbf{B}\cdot d\mathbf{a} = \int \nabla\cdot\mathbf{B}d^3\mathbf{r} = 0 \tag{5}$$

since $\nabla\cdot\mathbf{B} = 0$ always. Now suppose we divide this closed surface into two separate surfaces (labelled surface 1 and surface 2) by drawing some closed curve around the closed surface. Then we must have

$$\int_1 \mathbf{B}\cdot d\mathbf{a} = -\int_2 \mathbf{B}\cdot d\mathbf{a} \tag{6}$$

Now we can keep the curve fixed, and also keep one of the surfaces (say, surface 2) fixed, while varying surface 1. It's clear that $\int_1 \mathbf{B}\cdot d\mathbf{a}$ has to remain the same no matter what we define surface 1 to be, provided it has the same boundary curve. Thus the independence of the integral on the bounding curve is a consequence of **B** being divergenceless.

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