MAGNETOSTATICS - THE LORENTZ FORCE LAW

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The starting point in the study of magnetism is the Lorentz force law, which states that a charge q moving at velocity **v** through a magnetic field **B** feels a force of

$$\boldsymbol{F}=q\boldsymbol{v}\times\boldsymbol{B} \tag{1}$$

This law is not derived; it is merely an expression of what is observed in experiments.

Magnetism is an unusual force in several ways. First, it acts on a charge only if the charge is moving relative to the field. Second, it produces a force perpendicular both to the field and the direction of motion. Third, as a consequence of the second point, a magnetic force cannot do any work. This is because work is defined as the integral of $\boldsymbol{F}\cdot d\boldsymbol{\ell}$, in which only the component of force in the direction of motion appears. Since the magnetic force is always perpendicular to the direction of motion, $\boldsymbol{F}\cdot d\boldsymbol{\ell}=0$ always, so no work is done. The magnetic force can therefore change only the direction of motion, and not its speed.

As a simple example of this, suppose we send a charged particle into a region with a constant magnetic field. The particle's velocity is perpendicular to the field. The particle will feel a constant force of magnitude qvB perpendicular to its direction of motion, so that this force acts as a centripetal force, and the particle moves in a circle. Equating these two forces, we get

$$qvB=\frac{mv^2}{r} \tag{2}$$

This is known as the cyclotron formula, since it describes the principle used in a cyclotron, where charged particles are made to travel in circles by shooting them between the poles of a large electromagnet.

Using this formula, the radius of the circle is

$$r=\frac{mv}{qB} \tag{3}$$

The momentum of the particle can be expressed in terms of the radius, charge and field:

$$p = mv \quad (4)$$
$$= qrB \quad (5)$$

Suppose the field points along the $+y$ direction and the particle starts off moving in the $+x$ direction. After moving in the field a horizontal distance a, we find the particle is deflected in the $-z$ direction by a vertical distance d. By applying the right hand rule for cross products, we find that the particle must be positively charged. We can also work out its momentum (and thus its mass, assuming we measure its speed) if we can find the radius of the circular path.

If we draw a triangle with one vertex at the centre of the circle, another where the particle has reached its displacement of d below the xy plane, and then draw a horizontal line from the second vertex to intersect the z axis, we get a right angled triangle with sides of r, $r-d$ and a. From Pythagoras, we have

$$r^2 = (r-d)^2 + a^2 \quad (6)$$
$$0 = -2rd + d^2 + a^2 \quad (7)$$
$$r = \frac{a^2+d^2}{2d} \quad (8)$$

Therefore, the momentum is

$$p = qrB \quad (9)$$
$$= \frac{qB}{2d}\left(a^2+d^2\right) \quad (10)$$

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