MOTIONAL EMF

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Post date: 23 Mar 2021.

The most common type of electromotive force (emf) in practice is so-called *motional emf*, since the emf is generated by moving a wire through a magnetic field. This is the principle used by all electric power stations: some external force (hydro power to turn the generators, steam produced by burning some fuel such as coal or gas, or by nuclear reactions, turns the generators, wind power in wind turbines, and so on) is used to move the wires within a magnetic field, and electric current is produced.

The simplest example of motional emf is a rectangular loop of wire with one end inside a magnetic field and the other outside the field. If we pull the loop at velocity $\mathbf{v}$ so that the edge inside the field moves perpendicular to the field, then the Lorentz force law says that the force per unit charge is $\mathbf{f}_{\text{mag}}=\mathbf{v}\times\mathbf{B}$ and if we integrate this along the edge (of length ℓ, say) we get

$$\mathcal{E}=\oint\mathbf{f}_{\text{mag}}\cdot d\boldsymbol{\ell}=\ell Bv \tag{1}$$

Only one edge contributes to the integral, since the upper and lower edges are perpendicular to the force and the outside vertical edge is outside the field. This motional emf then generates a current according to Ohm's law: $I=\mathcal{E}/R=\ell Bv/R$, where R is the resistance in the wire.

However, once a current in the vertical direction starts flowing, there is a vertical component of velocity for the charges, which adds a component to the magnetic force. From the Lorentz force law, the force on the vertical segment of wire carrying a current I is

$$F=I\int|d\boldsymbol{\ell}\times\mathbf{B}|=I\ell B=\frac{\ell^2B^2v}{R} \tag{2}$$

where the direction of $\mathbf{F}$ is given by the right hand rule and the direction of the current, and will oppose the force pulling the wire through the magnetic field. Note that $\mathbf{F}$ is the *total* force on the wire due to the magnetic field acting on the overall current, while $\mathbf{f}_{\text{mag}}$ is only the force *per unit charge*, so we can't combine the two to get the total force on the wire unless we know the charge density within the wire.

Now suppose we have a loop composed of two parallel metal rails a distance ℓ apart connected at one end by a fixed wire containing a resistor R. A metal bar (perpendicular to the rails) is free to slide along the rails to form the other end of the loop. The bar starts off sliding at a velocity v_0. A uniform magnetic field **B** is applied over the entire loop, facing downwards. The motional emf generated by the bar's motion is then

$$\mathcal{E}=\ell Bv_0 \tag{3}$$

at the start. However, there will be a horizontal force opposing the bar's motion as given above. When the bar has slowed to a speed v the force is

$$F=-\frac{\ell^2B^2v}{R}=m\frac{dv}{dt} \tag{4}$$

where m is the mass of the bar, and we've introduced a minus sign to indicate that the bar is slowing down, so $dv/dt<0$. The solution of this is

$$v(t)=v_0e^{-t\ell^2B^2/mR} \tag{5}$$

Since the initial kinetic energy of the bar is $\frac{1}{2}mv_0^2$ this energy should be transferred as heat to the resistor by the time the bar comes to rest. We have

$$\begin{aligned} W &= \int_0^\infty I^2R\,dt && (6)\\ &= \frac{\ell^2B^2}{R}\int_0^\infty v^2(t)\,dt && (7)\\ &= \frac{\ell^2B^2v_0^2}{R}\int_0^\infty e^{-2t\ell^2B^2/mR}dt && (8)\\ &= \frac{1}{2}mv_0^2 && (9)\end{aligned}$$

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