OHM'S LAW, CONDUCTIVITY AND RESISTIVITY

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Up to now, our study of electromagnetism has concentrated on static charge distributions (electrostatics) or steady current distributions (giving rise to magnetostatics). Now we start looking in more detail at what happens when currents flow in arbitrary ways.

To begin, we can state the relation between electromagnetic force and current density:

$$\mathbf{J}=\sigma\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right) \tag{1}$$

where σ is the *conductivity* of the material in which the current is flowing. This is an empirical relation, based on observation, rather than a theoretically derived result. Note that σ here is *not* surface charge density; it is the standard notation for conductivity. Its inverse is $\rho=1/\sigma$, called the *resistivity*, and again should not be confused with volume charge density. Typically, the velocity $\mathbf{v}$ of the charges is so small that the magnetic force term can be neglected.

As an example, suppose we have two concentric conducting spheres of radii a and b with $b>a$, held at a potential difference of V. The area between the spheres is filled with a conducting material with conductivity σ. Our problem is to determine the current that flows between the spheres.

The electric field between the spheres is due entirely to the inner sphere, which we'll assume has a surface charge density of s. Then the field between the spheres is

$$\mathbf{E}=\frac{4\pi a^{2}s}{4\pi\epsilon_{0}}\frac{\hat{\mathbf{r}}}{r^{2}}=\frac{a^{2}s}{\epsilon_{0}}\frac{\hat{\mathbf{r}}}{r^{2}} \tag{2}$$

Ignoring the magnetic term, the current density is

$$\mathbf{J}=\sigma\mathbf{E}=\frac{a^{2}s\sigma}{\epsilon_{0}}\frac{\hat{\mathbf{r}}}{r^{2}} \tag{3}$$

and the total current between the spheres is

$$I=\int\mathbf{J}\cdot d\mathbf{a}=\frac{a^{2}s\sigma}{\epsilon_{0}}\int_{0}^{2\pi}\int_{0}^{\pi}\frac{r^{2}\sin\theta}{r^{2}}d\theta d\phi=\frac{4\pi a^{2}s\sigma}{\epsilon_{0}} \tag{4}$$

where the integral is done over a sphere of radius r such that $a<r<b$.

We still need to get rid of the explicit reference to the surface charge density s, which we can do by imposing the condition that the potential difference between the spheres is V. We consider a radial path from the outer to the inner shell, which gives

$$V=-\int_{b}^{a}\mathbf{E}\cdot d\ell=-\frac{sa^{2}}{\epsilon_{0}}\int_{b}^{a}\frac{dr}{r^{2}}=\frac{sa^{2}}{\epsilon_{0}}\left(\frac{1}{a}-\frac{1}{b}\right)=\frac{sa\,(b-a)}{\epsilon_{0}b} \tag{5}$$

Thus the surface charge density is

$$s=\frac{\epsilon_{0}bV}{a\,(b-a)} \tag{6}$$

and the current is

$$I=\frac{4\pi ab\sigma}{b-a}V \tag{7}$$

Note that the current is proportional to the potential difference, which is often true in resistive materials and is known as *Ohm's law,* which is more usually written as

$$V=IR \tag{8}$$

where R is the resistance of the material. In this case

$$R=\frac{b-a}{4\pi ab\sigma} \tag{9}$$

For large b, $I\to 4\pi a\sigma V$ and $R\to 1/4\pi a\sigma$. This is presumably because there is much more conducting material at a large radius, so it contributes less to the total resistance between the spheres.

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