POINT CHARGE IN HYPERBOLIC MOTION - VISIBLE AND INVISIBLE POINTS

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When we work out Liénard-Wiechert potentialsfor a moving point charge, we've been implicitly assuming that we can receive a signal from the charge from only one retarded time. That is, if the charge is moving on some trajectory $\mathbf{w}(t)$, there is only one point on that trajectory where a signal can be sent such that it will reach us at the time of observation. This assumption actually depends on special relativity with its postulate that nothing can travel faster than light. To see how this comes about, suppose there were, in fact, two different points P_1 and P_2 on the trajectory that could send signals that both arrived at the same time. Then if the distances to these two points are d_1 and d_2, the retarded times for these points are

$$t_1 = t - \frac{d_1}{c} \tag{1}$$

$$t_2 = t - \frac{d_{2,}}{c} \tag{2}$$

or, in terms of the distances

$$d_1 - d_2 = c(t_2 - t_1) \tag{3}$$

If the particle is closer to us at t_2, so that $d_2 < d_1$ then the difference $d_1 - d_2$ represents the amount by which the distance to the particle has decreased. According to 3, the speed at which the particle must move to cover this distance is

$$\frac{d_1 - d_2}{t_2 - t_1} = c \tag{4}$$

That is, the average speed in the radial direction has to be c. If the particle also has some transverse velocity on its trajectory, its total speed must be greater than c, which isn't allowed. (If the velocity is entirely radial, then the particle would have to be moving at exactly c, which also isn't allowed for any particle with rest mass.) Thus there can be at most one point where we receive a signal from a moving point charge.

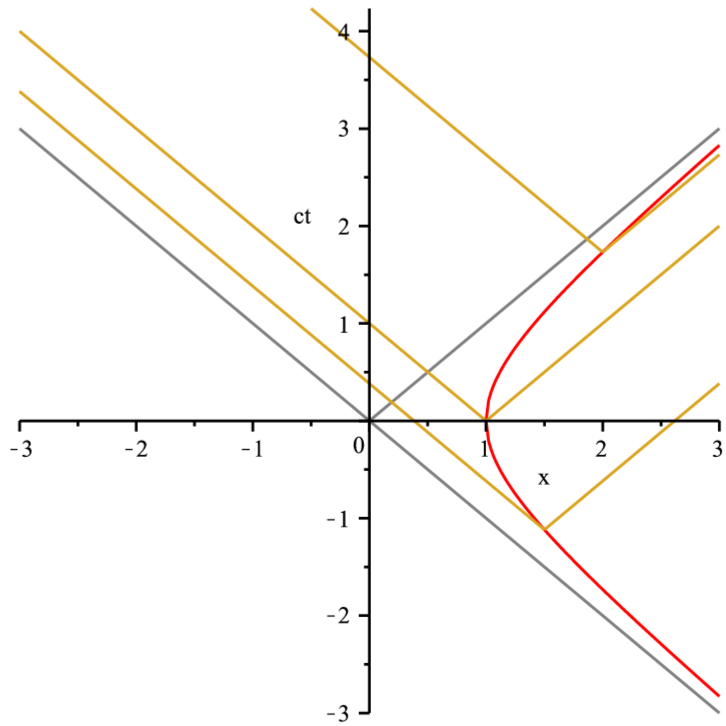

FIGURE 1. Point charge moving along trajectory $\mathbf{w}(t) = \sqrt{1+c^2t^2}\hat{\mathbf{x}}$ (red hyperbola).

It turns out that there are situations where a moving particle cannot be seen at all. For a simple example, suppose we have a particle moving along the x axis with a trajectory

$$\mathbf{w}(t) = \sqrt{b^2+c^2t^2}\hat{\mathbf{x}} \tag{5}$$

where $-\infty < t < +\infty$.

It's easiest to draw the trajectory on a spacetime diagram, with horizontal axis x and vertical axis ct, as usual. In Fig. 1, $b = 1$.

The red curve (a hyperbola) is the trajectory, so we see that the particle approaches from the right until it reaches closest approach to the origin at $x = 1$, then it moves away again. The grey lines are the asymptotes of the hyperbola, and are the lines $ct = \pm x$, so they represent photon paths. The yellow lines represent photons emitted by the particle at various points in its motion, and are parallel to the hyperbola's asymptotes. As usual, they move upwards to the left and right parallel to the lines $ct = \pm x$. We can see from the diagram that no photons can ever reach points below the line

$ct=-x$, so any observers in this region will be unaware of the particle's existence (so the potentials are zero in this area).

For a stationary observer at location x, his world line is a vertical line travelling upwards, so he will first see the particle when he crosses the line $ct=-x$. Once the particle becomes visible, it will remain visible forever, since the particle is visible everywhere above the line $ct=-x$.

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