POLARIZABILITY - SPHERICAL CHARGE DISTRIBUTION

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In an earlier post, we did a crude estimate of the separation between electron and proton in hydrogen due to polarization in an electric field. In that case, we assumed that the electron cloud was a uniformly charged sphere, and the result was that the induced dipole moment is proportional to the applied electric field.

Although this is also the result experimentally, it is possible to construct artificial cases where this relation isn't true. For example, suppose that the electron cloud is a sphere where the charge density is proportional to the distance from the centre out to a distance R. That is

$$\rho(r)=kr \tag{1}$$

We can find the total charge:

$$q = 4\pi\int_0^R kr\cdot r^2 dr \tag{2}$$

$$= \pi kR^4 \tag{3}$$

We can rewrite the charge density in terms of the total charge by eliminating k using 1:

$$\rho(r)=\frac{q}{\pi R^4}r \tag{4}$$

If the applied field separates the electron cloud and nucleus by a distance d then we can do the same calculation as in the earlier post to get a relation between the dipole moment and applied field. The nucleus must feel an equal and opposite field when it moves out the distance d. For a spherically symmetric charge distribution, the nucleus feels a force only from that portion of the sphere interior to the charge, so using Gauss's law we get

$$4\pi d^2 E = \frac{4\pi}{\epsilon_0}\int_0^d \frac{q}{\pi R^4} r\cdot r^2 dr \tag{5}$$

$$= \frac{qd^4}{\epsilon_0 R^4} \tag{6}$$

or

$$d = R^2\sqrt{\frac{4\pi\epsilon_0 E}{q}} \tag{7}$$

The dipole moment is $p = qd$ so we get

$$p = R^2\sqrt{4\pi\epsilon_0 qE} \tag{8}$$

In this case $p \propto E^{1/2}$.

If we have a radially symmetric charge density, we can work out the condition on $\rho(r)$ so that $p \propto E$. In general, from Gauss's law, we have for the field due to the portion of the sphere interior to $r = d$:

$$4\pi d^2 E = \frac{4\pi}{\epsilon_0}\int_0^d \rho(r)r^2 dr \tag{9}$$

$$E = \frac{1}{d^2\epsilon_0}\int_0^d \rho(r)r^2 dr \tag{10}$$

Since d is very small, we can expand $\rho(r)$ in the region $[0,d]$ in a Taylor series to get

$$\rho(r) = \rho(0) + r\rho'(0) + \ldots \tag{11}$$

In order for $p = qd \propto E$, the leading term in the integral must contain d^3, which means that we must have $\rho(0) \neq 0$. This is true of the uniformly charged sphere, but obviously not for the case in the current example.