RADIATION DAMPING

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Post date: 21 August 2021.

The radiation reaction force is given by the Abraham-Lorentz formula

$$F_{rad} = m\tau\dot{a} \tag{1}$$

where

$$\tau \equiv \frac{\mu_0 q^2}{6\pi mc} \tag{2}$$

The force gives rise to *radiation damping*. A simple example is that of a charge on a spring with natural frequency ω_0. If the charge is subject to a driving force that forces the charge to oscillate with frequency ω then from Newton's law

$$m\ddot{x} = F_{spring} + F_{rad} + F_{driving} \tag{3}$$

$$= -m\omega_0^2 x + m\tau\dddot{x} + F_{driving} \tag{4}$$

Since the charge is being forced to move sinusoidally with frequency ω we have

$$x(t) = x_0 \cos(\omega t + \delta) \tag{5}$$

where x_0 is the amplitude and δ is the phase. Thus

$$\dddot{x} = -x_0\omega^3 \sin(\omega t + \delta) \tag{6}$$

$$= -\omega^2 \dot{x} \tag{7}$$

We can rewrite 4 as

$$m\ddot{x} = -m\omega_0^2 x - m\tau\omega^2\dot{x} + F_{driving} \tag{8}$$

$$F_{driving} = m\ddot{x} + m\omega_0^2 x + m\tau\omega^2\dot{x} \tag{9}$$

Typically, a damping force (such as the drag force from travelling through a viscous fluid, as given by Stokes's law) is proportional to the velocity, so we can write the radiation damping term as

$$F_{rad} = m\tau\omega^2\dot{x} = m\gamma\dot{x} \tag{10}$$

where γ is the damping factor:

$$\gamma = \omega^2\tau \tag{11}$$

Example 1. If we return to the case of resonances in a dispersive medium, we can investigate the effect of radiation damping on the motion of an electron driven by an electromagnetic wave in the optical region. We found that the index of refraction in such a medium tends to increase with frequency, except near certain resonance frequencies where the index of refraction drops briefly before resuming its rise after the resonance frequency is passed. The width of this *anomalous dispersion* region is approximately $\Delta\omega \approx \gamma$, provided that $\gamma \ll \omega_0$ (the resonance frequency).

For an electron,

$$\tau = \frac{\left(4\pi\times10^{-7}\right)\left(1.6\times10^{-19}\right)^2}{6\pi\left(9.11\times10^{-31}\right)\left(3\times10^{8}\right)} = 6.24\times10^{-24}\text{s} \tag{12}$$

Taking the optical region to be green light, the frequency is about $\nu = 6\times10^{14}\text{s}^{-1}$, so $\omega = 2\pi\nu = 3.77\times10^{15}\text{s}^{-1}$, so

$$\gamma = 8.87\times10^{7}\text{s}^{-1} \tag{13}$$

This is much less than ω in the optical region.

Using the simple model of the hydrogen atom as an electron embedded in a uniform sphere of charge, we found that its resonant frequency was

$$\omega_0 = 4.13\times10^{16}\text{s}^{-1} \tag{14}$$

so $\gamma \ll \omega_0$ for an electron. [ω_0 here is in the near ultraviolet.] The width of the anomalous dispersion region in this model is therefore

$$\Delta\omega \approx \gamma = \omega_0^2\tau = 1.06\times10^{10}\text{s}^{-1} \tag{15}$$

Compared to the resonant frequency, $\Delta\omega$ is very small.