RADIATION REACTION - A FEW EXAMPLES

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Here are a few simple examples of the radiation reaction force, calculated using the Abraham-Lorentz formula.

Example 1. A charge q moves in a circle of radius R with constant speed v. The reaction force is given by

$$\mathbf{F}_{rad}=\frac{\mu_0 q^2}{6\pi c}\dot{\mathbf{a}} \tag{1}$$

For uniform circular motion, the acceleration is always directed towards the centre of the circle, so

$$\mathbf{a}=-\frac{v^2}{R}\hat{\mathbf{r}} \tag{2}$$
$$=-\frac{v^2}{R^2}\mathbf{r} \tag{3}$$
$$\dot{\mathbf{a}}=-\frac{v^2}{R^2}\mathbf{v} \tag{4}$$
$$\mathbf{F}_{rad}=-\frac{\mu_0 q^2 v^2}{6\pi c R^2}\mathbf{v} \tag{5}$$

The force we need to apply to counter the reaction force is just the negative of this, so

$$\mathbf{F}_e=\frac{\mu_0 q^2 v^2}{6\pi c R^2}\mathbf{v} \tag{6}$$

The power generated by applying this force is

$$P_e=\mathbf{F}_e\cdot\mathbf{v}=\frac{\mu_0 q^2 v^4}{6\pi c R^2} \tag{7}$$

The Larmor formula for radiated power is

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \tag{8}$$
$$= \frac{\mu_0 q^2 v^4}{6\pi c R^2} \tag{9}$$
$$= P_e \tag{10}$$

So the power we must exert to counter the reaction force is equal to the power radiated away.

Example 2. Now suppose we have a charge on a spring which moves with simple harmonic motion according to

$$\mathbf{r}(t) = A\hat{\mathbf{z}}\cos\omega t \tag{11}$$
$$\mathbf{v}(t) = -A\omega\hat{\mathbf{z}}\sin\omega t \tag{12}$$
$$\mathbf{a}(t) = -A\omega^2\hat{\mathbf{z}}\cos\omega t \tag{13}$$
$$\dot{\mathbf{a}}(t) = A\omega^3\hat{\mathbf{z}}\sin\omega t \tag{14}$$

The reaction force is now

$$\mathbf{F}_{rad} = \frac{\mu_0 q^2 A\omega^3}{6\pi c}\hat{\mathbf{z}}\sin\omega t \tag{15}$$

and the force $\mathbf{F}_e$ we need to apply to counter it is

$$\mathbf{F}_e = -\frac{\mu_0 q^2 A\omega^3}{6\pi c}\hat{\mathbf{z}}\sin\omega t \tag{16}$$

giving a power of

$$P_e = \mathbf{F}_e \cdot \mathbf{v} \tag{17}$$
$$= \frac{\mu_0 q^2 A^2\omega^4}{6\pi c}\sin^2\omega t \tag{18}$$

The radiated power is

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \tag{19}$$
$$= \frac{\mu_0 q^2 A^2\omega^4}{6\pi c}\cos^2\omega t \tag{20}$$

Thus in this case, the applied power is not equal to the radiated power at each instant of time, but remember that the Abraham-Lorentz formula was derived by taking the average power over a period of time after which the system returns to its initial state. If we average these two powers over one cycle, we get

$$\langle P_e \rangle = \frac{\mu_0 q^2 A^2 \omega^4}{6\pi c} \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \sin^2 \omega t \tag{21}$$

$$= \frac{\mu_0 q^2 A^2 \omega^4}{12\pi c} \tag{22}$$

$$= \langle P \rangle \tag{23}$$

Thus, on average, the powers are equal.

Example 3. For a charge falling in a constant gravitational field with acceleration g, $\dot{\mathbf{a}} = 0$ so the reaction force is zero. However, since the acceleration is not zero, the charge does radiate with a power of

$$P = \frac{\mu_0 q^2 g^2}{6\pi c} \tag{24}$$

In this case, there is no time interval after which the charge returns to its initial state, so we can't average the power over any time interval and the Abraham-Lorentz formula isn't valid.