RADIATION RESISTANCE

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The model of an oscillating dipole that we've been using consists of two charges exchanging charge by passing a current along a wire of length ℓ joining the charges. The average total power radiated by the dipole is

$$\langle P\rangle=\frac{\mu_0\omega^4p_0^2}{12\pi c} \tag{1}$$

where p_0 is the maximum dipole moment

$$p_0=q_0\ell \tag{2}$$

Here q_0 is the maximum charge at one end of the dipole, and the charge oscillates according to

$$q(t)=q_0\cos\omega t \tag{3}$$

If the energy lost through radiation were to be lost instead by heat generated by the current passing through the wire joining the charges, what would the resistance of the wire need to be? This resistance is known as the *radiation resistance*. The power generated by a current I passing through a resistor R is

$$P=I^2R \tag{4}$$

and in this case, the current is given by

$$I=\frac{dq}{dt}=-\omega q_0\sin\omega t \tag{5}$$

We're interested in the average power, so we want the average of I^2 over a single cycle. The average of $\sin^2 x$ over a cycle is $\frac{1}{2}$ so we get

$$\left\langle I^2\right\rangle=\frac{1}{2}\omega^2q_0^2 \tag{6}$$

$$\langle P\rangle=\frac{1}{2}\omega^2q_0^2R \tag{7}$$

Equating this to 1 we get

$$R = \frac{\mu_0 \omega^2 p_0^2}{6\pi c q_0^2} \tag{8}$$

$$= \frac{\mu_0 c \omega^2 q_0^2 \ell^2}{6\pi c^2 q_0^2} \tag{9}$$

$$= \frac{\mu_0 c}{6\pi}\left(\frac{2\pi}{\lambda}\right)^2 \ell^2 \tag{10}$$

$$= \frac{2\pi\mu_0 c}{3}\frac{\ell^2}{\lambda^2} \tag{11}$$

$$= 787\frac{\ell^2}{\lambda^2}\ \Omega \tag{12}$$

where in line 3 we used $\omega/c = 2\pi/\lambda$.

For a typical radio, we can take the length of a wire connecting components to be around $\ell = 5$ cm, while radio waves typically have wavelengths around 1 km, so the radiative resistance is around

$$R = 787\left(\frac{0.05}{10^3}\right)^2 = 2\times 10^{-6}\ \Omega \tag{13}$$

This is much smaller than typical resistances in a radio's circuits.