ANTICOMMUTATORS OF GAMMA MATRICES

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The gamma matrices used in calculations involving the Dirac equation are defined by

$$\begin{aligned}\gamma^0 &= \beta \\ \gamma^i &= \beta\alpha_i\end{aligned} \tag{1}$$

where the matrices are given by

$$\alpha_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

$$\alpha_2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

$$\alpha_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \tag{4}$$

$$\beta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{5}$$

These matrices satisfy the conditions

$$\beta^2 = \alpha_i^2 = I \tag{6}$$

$$\{\beta, \alpha_i\} = 0 \tag{7}$$

$$\{\alpha_i, \alpha_k\} = 2I\delta_{ik} \tag{8}$$

where I is the 4×4 identity matrix, and the braces $\{\}$ indicate an anticommutator, so that

$$\{\beta,\alpha_i\}\equiv\beta\alpha_i+\alpha_i\beta \tag{9}$$

and so on.

Using these definitions and properties, we can work out the anticommutators of the gamma matrices. Consider first the spatial components. We have

$$\begin{aligned}\left\{\gamma^i,\gamma^k\right\} &= \gamma^i\gamma^k+\gamma^k\gamma^i && (10)\\ &= \beta\alpha_i\beta\alpha_k+\beta\alpha_k\beta\alpha_i && (11)\\ &= \beta\left(-\beta\alpha_i\right)\alpha_k+\beta\left(-\beta\alpha_k\right)\alpha_i && (12)\\ &= -\beta^2\left(\alpha_i\alpha_k+\alpha_k\alpha_i\right) && (13)\\ &= -I\left\{\alpha_i,\alpha_k\right\} && (14)\\ &= -2I\delta_{ik} && (15)\end{aligned}$$

Anticommutators involving γ^0 can be worked out similarly. We have

$$\begin{aligned}\left\{\gamma^0,\gamma^i\right\} &= \gamma^0\gamma^i+\gamma^i\gamma^0 && (16)\\ &= \beta\beta\alpha_i+\beta\alpha_i\beta && (17)\\ &= I\alpha_i-\beta^2\alpha_i && (18)\\ &= I\left(\alpha_i-\alpha_i\right) && (19)\\ &= 0 && (20)\end{aligned}$$

Finally, we have

$$\left\{\gamma^0,\gamma^0\right\}=2\left(\gamma^0\right)^2=2\beta^2=2I \tag{21}$$

We can combine 15, 20 and 21 using the metric tensor

$$g^{\mu\nu}=\text{diag}\left(1,-1,-1,-1\right) \tag{22}$$

That is, $g^{\mu\nu}$ is the 4×4 diagonal matrix with the diagonal elements indicated. We have

$$\left\{\gamma^\mu,\gamma^\nu\right\}=2g^{\mu\nu} \tag{23}$$

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