COMPTON SCATTERING WITH FEYNMAN RULES

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As an example of evaluating an S matrix element using Feynman rules, we'll consider Compton scattering, which is the scattering of a photon by an electron. There are two second-order Feynman diagrams that describe this process, as in Fig. 1.

The first Feynman rule gives us the factor

$$(2\pi)^4\delta\left(P_f-P_i\right)\prod_{\text{ext bosons}}\frac{1}{\sqrt{2V\omega_{\boldsymbol{k}}}}\prod_{\text{ext fermions}}\sqrt{\frac{m}{VE_{\boldsymbol{p}}}} \tag{1}$$

In this case, there are two external fermions (both electrons) and two external photons, so we have

$$(2\pi)^4\delta\left(p'+k'-p-k\right)\frac{1}{\sqrt{2V\omega_{\boldsymbol{k}}}}\frac{1}{\sqrt{2V\omega_{\boldsymbol{k}'}}}\sqrt{\frac{m}{VE_{\boldsymbol{p}}}}\sqrt{\frac{m}{VE_{\boldsymbol{p}'}}} \tag{2}$$

This factor is common to both diagrams.

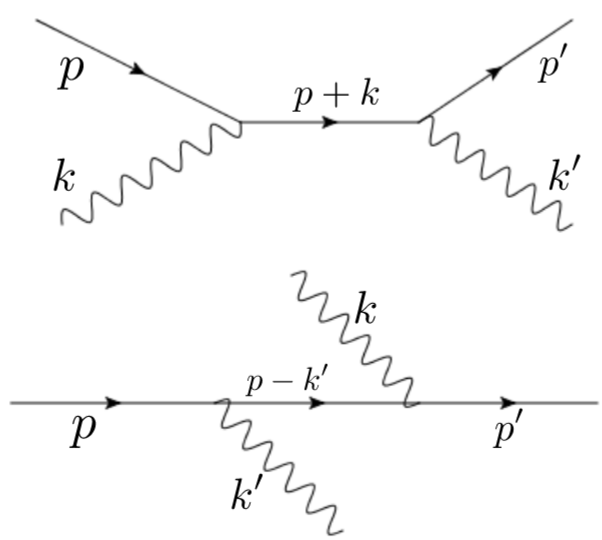

FIGURE 1. Second-order Feynman diagrams for Compton scattering.

initial electron	$u_r(\boldsymbol{p})$
final electron	$\bar{u}_r(\boldsymbol{p})$
initial positron	$\bar{v}_r(\boldsymbol{p})$
final positron	$v_r(\boldsymbol{p})$
initial photon	$\varepsilon_{\mu,r}(\boldsymbol{k})$
final photon	$\varepsilon_{\mu,r}(\boldsymbol{k})$

TABLE 1. External Feynman factors.

For the top diagram in Fig. 1, we proceed as follows. We add the factors as described by Table 1.

There is one incoming electron, so we have $u_r(\boldsymbol{p})$. We then have a vertex, so we have a factor of $ie\gamma^\mu$ (rule 1). Then we have an incoming photon, so we have a factor of $\varepsilon_{\mu,s}$. This gives us the first vertex factor of

$$\varepsilon_{\mu,s}(\boldsymbol{k})\, ie\gamma^\mu u_r(\boldsymbol{p}) \tag{3}$$

Then we have an internal fermion line with momentum $p+k$, so we have a factor of $iS_F(p+k)$ (rule 3). Finally, we have an outgoing fermion giving a factor of $\bar{u}_{r'}(\boldsymbol{p}')$, another factor of $ie\gamma^\nu$ for the vertex, and a factor of $\varepsilon_{\nu,s'}(\boldsymbol{k}')$ for the outgoing photon. Putting it all together, we have the contribution to $\mathcal{M}^{(2)}$ from the top diagram in Fig. 1:

$$\mathcal{M}^{(2)}_{\text{top}} = \varepsilon_{\nu,s'}(\boldsymbol{k}')\, ie\bar{u}_{r'}(\boldsymbol{p}')\, \gamma^\nu iS_F(p+k)\, \varepsilon_{\mu,s}(\boldsymbol{k})\, ie\gamma^\mu u_r(\boldsymbol{p}) \tag{4}$$

$$= -e^2 \varepsilon_{\nu,s'}(\boldsymbol{k}')\, \bar{u}_{r'}(\boldsymbol{p}')\, \gamma^\nu iS_F(p+k)\, \varepsilon_{\mu,s}(\boldsymbol{k})\, \gamma^\mu u_r(\boldsymbol{p}) \tag{5}$$

Note the order of the various factors. The product $\bar{u}_{r'}(\boldsymbol{p}')\gamma^\nu$ is of a row vector with a 4×4 matrix, which produces a row vector. The product $\gamma^\mu u_r(\boldsymbol{p})$ produces a column vector, so the overall product $\bar{u}_{r'}(\boldsymbol{p}')\gamma^\nu\gamma^\mu u_r(\boldsymbol{p})$ produces a single number. This number is different for each combination of μ,ν, but these are summed over by multiplying by the ε terms. Thus the overall result is a single number.

For the bottom diagram, we apply the rules to get

$$\mathcal{M}^{(2)}_{\text{bottom}} = \varepsilon_{\nu,s}(\boldsymbol{k})\, ie\bar{u}_{r'}(\boldsymbol{p}')\, \gamma^\nu iS_F(p-k')\, \varepsilon_{\mu,s'}(\boldsymbol{k}')\, ie\gamma^\mu u_r(\boldsymbol{p}) \tag{6}$$

$$= -e^2 \varepsilon_{\nu,s}(\boldsymbol{k})\, \bar{u}_{r'}(\boldsymbol{p}')\, \gamma^\nu iS_F(p-k')\, \varepsilon_{\mu,s'}(\boldsymbol{k}')\, \gamma^\mu u_r(\boldsymbol{p}) \tag{7}$$

Combining with 2, we get the total second-order S matrix element.

$$S^{(2)}=\left[(2\pi)^{4}\delta\left(p'+k'-p-k\right)\frac{m}{2V^{2}\sqrt{\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}E_{\boldsymbol{p}}E_{\boldsymbol{p}'}}}\right]\left(\mathcal{M}_{\text{top}}^{(2)}+\mathcal{M}_{\text{bottom}}^{(2)}\right) \tag{8}$$

This agrees with Klauber's result (done the long way, without using Feynman rules) in his equation (8-67).