DECAY OF A SINGLE PARTICLE

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In Chapter 12, Coleman uses his formulas from Chapter 11 to calculate the probabilities of several processes. The first case he considers is that of the decay of a single particle into an arbitrary number of outgoing particles. To review, we shall use the expressions

$$\frac{\text{differential transition prob}}{\text{unit time}} = \left|\mathcal{A}_{fi}\right|^2 D \prod_{\text{initial}} \frac{1}{2E_i} \tag{1}$$

$$D \equiv (2\pi)^4 \delta^{(4)}\left(p_f - p_i\right) \prod_{\text{final}} \frac{d^3\boldsymbol{p}_f}{(2\pi)^3 2E_f} \tag{2}$$

The amplitude $\mathcal{A}_{fi}$ is the invariant Feynman amplitude, whose form depends on the particular model we are studying. For the hamiltonian

$$H_I = g\psi^\dagger\psi\phi \tag{3}$$

that we've been studying so far, we have, at second order:

$$\mathcal{A}_{fi} \equiv \frac{(-ig)^2}{\left(p_1 - p_1'\right)^2 - \mu^2 + i\epsilon} + \frac{(-ig)^2}{\left(p_1 - p_2'\right)^2 - \mu^2 + i\epsilon} \tag{4}$$

For other processes, $\mathcal{A}_{fi}$ will have a different form, but if we apply the Feynman rules, there will always be terms containing squares of four-momenta, which are Lorentz invariant quantities.

We note that both D and $\mathcal{A}_{fi}$ are Lorentz invariant quantities. This is true for D because the term in the product is the Lorentz invariant integration measure. It is true for $\mathcal{A}_{fi}$ because 4 contains only the squares of 4-vectors (and a couple of scalars). Thus the only term in 1 that is *not* Lorentz invariant is the factor $\frac{1}{2E_i}$ in the product over initial states. This isn't invariant since the energy of the incoming particles depends on their speeds (as viewed by some observer), so E_i is larger for particles moving more quickly.

To apply these formulas to the decay of a single particle, Coleman considers the particle to be at rest in the lab frame, so that its energy is

$$E_i = m \tag{5}$$

where m is its rest mass.

To find the total decay probability per unit time, we need to sum over all possible final states (that is, if the particle can decay in several different ways, we need to sum over all these possibilities). We also need to sum over all possible final momenta. Due to energy conservation, only those collections of final momenta that are equal to the particle's initial energy (that is, its rest mass m) are possible. This restriction is enforced by the delta function in 2. [If the presence of a delta function in an expression for the probability of something happening bothered you (as it did me) because a delta function has an infinity implicit in it, this problem is resolved by the fact that the delta function $\delta^{(4)}\left(p_f - p_i\right)$ appears only when linked to a product of differentials $\prod_{\text{final}} \frac{d^3\boldsymbol{p}_f}{(2\pi)^3 2E_f}$. When an integration over these differentials is done, the delta function disappears so the final answer for the probability is finite.]

For the decay of a single particle, Coleman defines the differential decay probability per unit time with symbol $d\Gamma$. Since there is only one initial particle and we're considering the frame in which the particle is at rest, we have from 1

$$d\Gamma = \frac{\text{diff. decay prob.}}{\text{unit time}} = \frac{1}{2m}\left|\mathcal{A}_{fi}\right|^2 D \tag{6}$$

The total decay probability per unit time is the sum and integral of $d\Gamma$, so we have

$$\Gamma = \frac{1}{2m}\sum_{\text{final states}}\int \left|\mathcal{A}_{fi}\right|^2 D \tag{7}$$

If we know the physical quantities (g and m in the above model) we can plug these into 7 and, after all this theory, finally get an actual number for a transition probability, since all other quantities (the momenta) are integrated and summed, so don't appear in the final result. Thus we have finally produced something that can be compared with experiment.

One final comment is worth noting. The quantity m in 7 is the rest mass of the decaying particle, and appears because we're considering the initial particle to be at rest. If it were moving, then we would replace m by $E_i = \gamma m$ where γ is the usual relativistic factor $1/\sqrt{1-v^2}$. Thus a moving particle would have a value of Γ that is smaller by the factor γ. That is, its probability of decay per unit time is smaller than for a particle at rest, or in other words, its lifetime is longer, on average, and by the factor γ which is the usual time dilation factor. This effect is one of the early tests of special relativity, in which it was observed that the decay rate of particles produced by cosmic rays interacting with the Earth's atmosphere had a longer lifetime than those at rest.