

## FEYNMAN PROPAGATOR FOR SCALAR FIELDS

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In quantum field theory, interactions between particles are mediated by *virtual particles*, which are particles that are never observed, but which carry the information from the particles before the interaction to the particles after the interaction. A particle interaction can be represented graphically by a Feynman diagram. One example is Fig. 1.

In the diagram, time increases from left to right. The arrow on a line indicates the motion of a particle, and rather confusingly at first, antiparticles are shown with arrows opposite to their direction of propagation. Thus in this interaction, an electron  $e^-$  and a positron (antielectron)  $e^+$  move in from the left and meet at point  $x_2$ . They annihilate each other, producing a photon (the wavy line, labelled  $\gamma$  for gamma particle). The photon is a virtual particle (it is never observed in an experiment) which propagates to location  $x_1$  where it spontaneously dissociates into an electron-positron pair which move off to the right.

The physics of the virtual particle, the photon in this case, is described by a *Feynman propagator*, or just *propagator*. In its simplest form, a propagator creates a virtual particle from the vacuum and, a short time later, annihilates it. We can use the Klein-Gordon fields derived earlier to see how this works. [Note that a photon is not described by a Klein-Gordon field, since the photon has spin 1 and is not a scalar particle.]

The continuous fields are

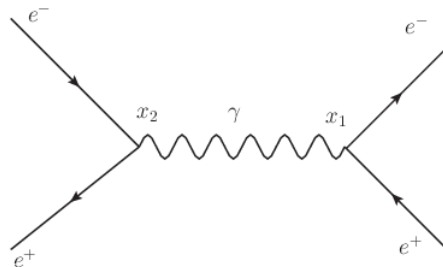
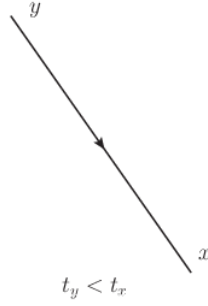
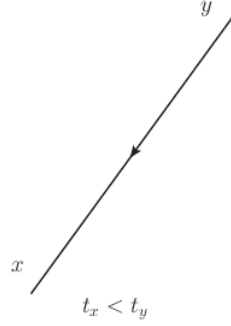


FIGURE 1. Feynman diagram for  $e^+ - e^-$  interaction.

FIGURE 2. Particle created at  $y$  and annihilated at  $x$ .FIGURE 3. Antiparticle created at  $x$  and annihilated at  $y$ .

$$\phi(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3 \omega_{\mathbf{k}}}} a(\mathbf{k}) e^{-ikx} + \int \frac{d^3k}{\sqrt{2(2\pi)^3 \omega_{\mathbf{k}}}} b^\dagger(\mathbf{k}) e^{ikx} \quad (1)$$

$$\equiv \phi^+ + \phi^- \quad (2)$$

$$\phi^\dagger(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3 \omega_{\mathbf{k}}}} b(\mathbf{k}) e^{-ikx} + \int \frac{d^3k}{\sqrt{2(2\pi)^3 \omega_{\mathbf{k}}}} a^\dagger(\mathbf{k}) e^{ikx} \quad (3)$$

$$\equiv \phi^{\dagger+} + \phi^{\dagger-} \quad (4)$$

For a scalar field, there are two situations. First, we can create a particle at location  $\mathbf{y}$  at time  $t_y$ , then at a later time  $t_x > t_y$ , we can annihilate the same particle at location  $\mathbf{x}$ . This is shown in Fig. 2

Second, if  $t_x < t_y$  we can create an antiparticle at  $\mathbf{x}$  and annihilate it at  $\mathbf{y}$ , as shown in Fig. 3.

The important point is that these two virtual particle situations have the same result in an experiment. If a virtual particle is created at  $\mathbf{y}$  at  $t_y$  and travels to  $\mathbf{x}$  at time  $t_x$ , it carries information (charge and so on) from  $\mathbf{y}$  to  $\mathbf{x}$ . If the corresponding virtual *antiparticle* is created at  $\mathbf{x}$  at  $t_x$  and travels to  $\mathbf{y}$  at  $t_y$ , it carries exactly the opposite information (since it's an antiparticle) from  $\mathbf{x}$  to  $\mathbf{y}$ . Thus in a real experiment, the propagator must include both possibilities.

Klauber treats the case of  $t_y < t_x$ , so we'll look at the other case (the two derivations are very similar). That is, we want to create an antiparticle at  $x$  and annihilate it at  $y$ . From the equations above, we see that  $\phi$  creates antiparticles (it contains the  $b^\dagger$  operators) and  $\phi^\dagger$  destroys them (it contains the  $b$  operators), so the life of the virtual particle is described by applying these two fields in some order. Since we want to create an antiparticle first and *then* annihilate it, we need to apply  $\phi$  first, then  $\phi^\dagger$ . The situation is reversed if we want to create a particle and then annihilate it, since in that case  $\phi^\dagger$  contains the  $a^\dagger$  operators and  $\phi$  contains the  $a$  operators. The two time orders above thus require the fields to be applied in different orders, and a *time ordering operator*  $T$  is defined so that

$$T \left[ \phi(x) \phi^\dagger(y) \right] = \begin{cases} \phi(x) \phi^\dagger(y) & \text{if } t_y < t_x \\ \phi^\dagger(y) \phi(x) & \text{if } t_x < t_y \end{cases} \quad (5)$$

We're interested in the second case. The *transition amplitude* for a process in which a virtual particle is created out of the vacuum and then decays back into the vacuum is then

$$\langle 0 | T \left[ \phi(x) \phi^\dagger(y) \right] | 0 \rangle \quad (6)$$

Looking at the antiparticle case, we have

$$T \left[ \phi(x) \phi^\dagger(y) \right] | 0 \rangle = \left( \phi^{\dagger+}(y) + \phi^{\dagger-}(y) \right) \left( \phi^+(x) + \phi^-(x) \right) | 0 \rangle \quad (7)$$

If we're looking at an antiparticle with a specific wave number  $\mathbf{k}$ , then  $\phi^-$  creates an antiparticle and  $\phi^{\dagger+}$  destroys an antiparticle (while  $\phi^+$  destroys a particle and  $\phi^{\dagger-}$  creates a particle). Any annihilation operator acting on the vacuum gives zero, so

$$\left( \phi^+(x) + \phi^-(x) \right) | 0 \rangle = \left( 0 + \phi^-(x) \right) | 0 \rangle \quad (8)$$

$$= A(x) | \bar{\phi} \rangle \quad (9)$$

where  $A(x)$  is a numerical function (not an operator), since a creation operator acting on the vacuum gives a number (determined by normalization) multiplied by the state  $|\bar{\phi}\rangle$  containing a single antiparticle.

Returning to 7, we see that operating on this result with  $\phi^{\dagger-}(y)$  creates a particle, so gives the state  $|\bar{\phi}\phi\rangle$  multiplied by some other numerical function  $B(y)$ , while operating on  $A(x)|\bar{\phi}\rangle$  with  $\phi^{\dagger+}(y)$  destroys the antiparticle just created, producing the vacuum state  $|0\rangle$  multiplied by some other numerical function  $C(y)$ . Therefore we get

$$T\left[\phi(x)\phi^{\dagger}(y)\right]|0\rangle = C(y)A(x)|0\rangle + B(y)A(x)|\bar{\phi}\phi\rangle \quad (10)$$

Thus the transition amplitude 6 is

$$\langle 0|T\left[\phi(x)\phi^{\dagger}(y)\right]|0\rangle = \langle 0|C(y)A(x)|0\rangle + \langle 0|B(y)A(x)|\bar{\phi}\phi\rangle \quad (11)$$

The brackets imply an integration over all space, but we're interested in the antiparticle creation occurring at a specific location  $x$  and annihilation at another specific location  $y$ , so these two locations are actually constants relative to the integration variable, and can come outside the brackets. From the orthonormality of quantum states,  $\langle 0|0\rangle = 1$  and  $\langle 0|\bar{\phi}\phi\rangle = 0$ , so we get

$$\langle 0|T\left[\phi(x)\phi^{\dagger}(y)\right]|0\rangle = C(y)A(x) \quad (12)$$

The result for creating and annihilating a particle (as opposed to an antiparticle) is the same, although the numerical functions can be different. Klauber calls them  $G(x)$  and  $F(y)$ , so that for the particle case

$$\langle 0|T\left[\phi(x)\phi^{\dagger}(y)\right]|0\rangle = F(y)G(x) \quad (13)$$

The vacuum expectation value of the time ordering operator is called the *Feynman propagator*, defined as  $i\Delta_F(x-y)$ :

$$i\Delta_F(x-y) \equiv \langle 0|T\left[\phi(x)\phi^{\dagger}(y)\right]|0\rangle \quad (14)$$

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