FEYNMAN PROPAGATOR IN TERMS OF COMMUTATORS

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We've seen that the creation and annihilation of a virtual particle or antiparticle is described by the Feynman propagator, defined as

$$i\Delta_F(x-y) \equiv \left\langle 0\left|T\left[\phi(x)\phi^\dagger(y)\right]\right|0\right\rangle \tag{1}$$

where T is the time ordering operator, which places its arguments (the fields) in the correct order to create and annihilate the particle or antiparticle. For the case where an antiparticle is created at location $\mathbf{x}$ at time t_x and annihilated at location $\mathbf{y}$ at time $t_y > t_x$, we have

$$T\left[\phi(x)\phi^\dagger(y)\right]|0\rangle = \left(\phi^{\dagger+}(y)+\phi^{\dagger-}(y)\right)\left(\phi^+(x)+\phi^-(x)\right)|0\rangle \tag{2}$$

where the continuous fields are defined by

$$\begin{aligned}
\phi(x) &= \int\frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\mathbf{k}}}}a(\mathbf{k})e^{-ikx}+\int\frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\mathbf{k}}}}b^\dagger(\mathbf{k})e^{ikx} &(3)\\
&\equiv \phi^+ + \phi^- &(4)\\
\phi^\dagger(x) &= \int\frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\mathbf{k}}}}b(\mathbf{k})e^{-ikx}+\int\frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\mathbf{k}}}}a^\dagger(\mathbf{k})e^{ikx} &(5)\\
&\equiv \phi^{\dagger+} + \phi^{\dagger-} &(6)
\end{aligned}$$

The field ϕ^+ annihilates the vacuum in 2, so produces zero. ϕ^- creates an antiparticle from the vacuum and the subsequent action of $\phi^{\dagger-}$ creates a particle, giving the state $\left|\phi\bar{\phi}\right\rangle$ (multiplied by some numerical factor). This state is orthogonal to the vacuum state $|0\rangle$ so drops out of the calculation in 1. This leaves only the state produced by the operation of ϕ^- followed by $\phi^{\dagger+}$, which creates and then annihilates an antiparticle. Thus

$$i\Delta_F(x-y) = \left\langle 0\left|\phi^{\dagger+}(y)\phi^-(x)\right|0\right\rangle \tag{7}$$

If we operated on the vacuum state with these field operators in the opposite order, we would get zero since $\phi^{\dagger+}(y)\left|0\right\rangle = 0$. Therefore we can replace the fields in 7 by

$$i\Delta_F(x-y) = \left\langle 0\left|\phi^{\dagger+}(y)\phi^{-}(x) - \phi^{-}(x)\phi^{\dagger+}(y)\right|0\right\rangle \tag{8}$$

$$= \left\langle 0\left|\left[\phi^{\dagger+}(y),\phi^{-}(x)\right]\right|0\right\rangle \tag{9}$$

Klauber does the equivalent calculation for a particle created at t_y and annihilated at $t_x > t_y$, with the result

$$i\Delta_F(x-y) = \left\langle 0\left|\left[\phi^{+}(x),\phi^{\dagger-}(y)\right]\right|0\right\rangle \tag{10}$$

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