FEYNMAN RULES FOR QED DIAGRAMS WITHOUT LOOPS

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When evaluating terms in the S matrix expansion, we are usually faced with a lot of work involving integrals over spacetime or 4-momentum. Fortunately there is a set of rules developed by Feynman which automates the process in most cases.

The complete set of Feynman rules deals with all types of Feynman diagrams, of all orders. Here, we'll look at the most basic set of rules, which apply to Feynman diagrams without loops. In QED, this includes diagrams with fermions (typically electrons and positrons) and photons.

The rules are:

The S matrix element S_{fi} from an initial state $|i\rangle$ to a final state $|f\rangle$ is given by

$$S_{fi} = \delta_{fi} + \left[(2\pi)^4 \delta\left(P_f - P_i\right) \prod_{\text{ext bosons}} \frac{1}{\sqrt{2V\omega_{\boldsymbol{k}}}} \prod_{\text{ext fermions}} \sqrt{\frac{m}{VE_{\boldsymbol{p}}}}\right] \mathcal{M} \tag{1}$$

where

$$\mathcal{M} \equiv \sum_{n=1}^{\infty} \mathcal{M}^{(n)} \tag{2}$$

and $\mathcal{M}^{(n)}$ is the total contribution from nth order Feynman diagrams, that is, diagrams containing n vertices.

The Feynman rules pertain to building $\mathcal{M}^{(n)}$. For QED diagrams without loops, they are

1. For each vertex, include a factor $ie\gamma^{\mu}$.

2. For each internal photon line (that is, a virtual photon), labelled by 4-momentum k, include a factor $iD_{F\mu\nu}(k)$, which is the photon propagator.

3. For each internal (virtual) fermion line with momentum p, include a factor $iS_F(p)$, which is the fermion propagator.

4. For each external line, include the corresponding factor as shown in Table 1.

5. The 4-momenta at each vertex must be conserved.

initial electron	$u_r(\boldsymbol{p})$
final electron	$\bar{u}_r(\boldsymbol{p})$
initial positron	$\bar{v}_r(\boldsymbol{p})$
final positron	$v_r(\boldsymbol{p})$
initial photon	$\varepsilon_{\mu,r}(\boldsymbol{k})$
final photon	$\varepsilon_{\mu,r}(\boldsymbol{k})$

TABLE 1. External Feynman factors.

6. The factors are ordered so that, reading from right to left, they follow the sequence reading along the corresponding line in the diagram. This is important for fermion factors, since they must be in the correct order so that the matrix multiplications of the spinors with the gamma matrices makes sense.

7. If more than one diagram applies to a given process (for example, there are two diagrams in electron-electron scattering), the fermion factors must be in the same order in all diagrams. If this requires swapping fermion operators in one of the diagrams, introduce a factor of -1 for each swap.

Some examples of applying the Feynman rules will follow in future posts.

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