

GENERATORS OF THE TRANSLATION OPERATOR

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the auxiliary blog and include the title or URL of this post in your comment.

Post date: 6 August 2023.

We've seen earlier that for an infinitesimal translation ε of the coordinate x , the translation operator is given by (in natural units so that $\hbar = 1$):

$$U(\varepsilon) = I - i\varepsilon P \quad (1)$$

where P is the momentum operator. For a finite translation a , this operator becomes

$$U(a) = e^{-iaP} \quad (2)$$

or, for a 3-d translation \mathbf{a} , and using hats to indicate explicitly which items are operators

$$\hat{U}(\mathbf{a}) = e^{-i\mathbf{a}\cdot\hat{\mathbf{P}}} \quad (3)$$

From this last form, we can regain the generator as follows:

$$\left. \frac{\partial \hat{U}(\mathbf{a})}{\partial \mathbf{a}} \right|_{\mathbf{a}=0} = -i\hat{\mathbf{P}} e^{-i\mathbf{a}\cdot\hat{\mathbf{P}}} \Big|_{\mathbf{a}=0} \quad (4)$$

$$= -i\hat{\mathbf{P}} \quad (5)$$

Thus the generator is given by

$$\hat{\mathbf{P}} = -\frac{1}{i} \left. \frac{\partial \hat{U}(\mathbf{a})}{\partial \mathbf{a}} \right|_{\mathbf{a}=0} \quad (6)$$