HEISENBERG PICTURE

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In the Heisenberg picture of quantum mechanics, the states are time-independent, with all the time dependence being shifted to the operators. That is, we can write (where a subscript H stands for 'Heisenberg')

$$\left|\psi(t)\right\rangle_H = \left|\psi(0)\right\rangle_H \tag{1}$$

for all t, so in fact we can just drop the t dependence from a Heisenberg state. Usually the Heisenberg state is taken to be the Schrödinger state (denoted with a subscript S) at $t=0$, so that

$$\left|\psi\right\rangle_H = \left|\psi(0)\right\rangle_S \tag{2}$$

If we start with a Schrödinger state at some time t, we can apply the time evolution equation

$$U\left(t,t'\right) = e^{-iH(p_S,q_S)(t-t')} \tag{3}$$

in reverse to de-evolve the state back to $t=0$, so we have

$$\left|\psi\right\rangle_H = e^{iH(p_S,q_S)t}\left|\psi(t)\right\rangle_S \tag{4}$$

The fundamental position and momentum operators are now defined to be time dependent. In the Schrödinger picture, the expectation value of q_S is obtained from the square modulus of $\left\langle\psi(t)\left|q_S\right|\psi(t)\right\rangle$. If we want to transfer the time dependence to the operator, we have

$$\begin{aligned}
{}_S\left\langle\psi(t)\left|q_S\right|\psi(t)\right\rangle_S &= {}_S\left\langle U(t,0)\psi(0)\left|q_S\right|U(t,0)\psi(0)\right\rangle_S && (5)\\
&= {}_S\left\langle\psi(0)\left|U^\dagger(t,0)\,q_S U(t,0)\right|\psi(0)\right\rangle_S && (6)\\
&= {}_H\left\langle\psi\left|U^\dagger(t,0)\,q_S U(t,0)\right|\psi\right\rangle_H && (7)
\end{aligned}$$

where we used 2 to get the last line. We can now identify the operator in the middle with the Heisenberg position operator, which is now a function of time:

$$q_H(t) = U^\dagger(t,0)\, q_S U(t,0) \tag{8}$$

A similar equation holds for p_H:

$$p_H(t) = U^\dagger(t,0)\, p_S U(t,0) \tag{9}$$

The Heisenberg and Schrödinger operators are equal at $t = 0$:

$$q_H(0) = q_S \tag{10}$$

$$p_H(0) = p_S \tag{11}$$

We can follow a similar procedure to convert any operator from the Schrödinger to the Heisenberg picture, as Coleman does in eqn 7.15. Note that a general operator $A_S(t)$ *may* have an explicit time dependence (for example, a Hamiltonian $H(p_S, q_S, t)$), so the general rule is

$$A_H(t) = U^\dagger(t,0)\, A_S(t)\, U(t,0) \tag{12}$$

$$= U(0,t)\, A_S(t)\, U^\dagger(0,t) \tag{13}$$

where the last line follows from

$$U\left(t',t\right) = U^{-1}\left(t,t'\right) = U^\dagger\left(t,t'\right) \tag{14}$$

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