

HEISENBERG PICTURE

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog and include the title or URL of this post in your comment.

Post date: 14 August 2021.

In the Heisenberg picture of quantum mechanics, the states are time-independent, with all the time dependence being shifted to the operators. That is, we can write (where a subscript H stands for 'Heisenberg')

$$|\psi(t)\rangle_H = |\psi(0)\rangle_H \quad (1)$$

for all t , so in fact we can just drop the t dependence from a Heisenberg state. Usually the Heisenberg state is taken to be the Schrödinger state (denoted with a subscript S) at $t = 0$, so that

$$|\psi\rangle_H = |\psi(0)\rangle_S \quad (2)$$

If we start with a Schrödinger state at some time t , we can apply the time evolution equation

$$U(t, t') = e^{-iH(p_S, q_S)(t-t')} \quad (3)$$

in reverse to de-evolve the state back to $t = 0$, so we have

$$|\psi\rangle_H = e^{iH(p_S, q_S)t} |\psi(t)\rangle_S \quad (4)$$

The fundamental position and momentum operators are now defined to be time dependent. In the Schrödinger picture, the expectation value of q_S is obtained from the square modulus of $\langle \psi(t) | q_S | \psi(t) \rangle$. If we want to transfer the time dependence to the operator, we have

$${}_S \langle \psi(t) | q_S | \psi(t) \rangle_S = {}_S \langle U(t, 0) \psi(0) | q_S | U(t, 0) \psi(0) \rangle_S \quad (5)$$

$$= {}_S \langle \psi(0) | U^\dagger(t, 0) q_S U(t, 0) | \psi(0) \rangle_S \quad (6)$$

$$= {}_H \langle \psi | U^\dagger(t, 0) q_S U(t, 0) | \psi \rangle_H \quad (7)$$

where we used 2 to get the last line. We can now identify the operator in the middle with the Heisenberg position operator, which is now a function of time:

$$q_H(t) = U^\dagger(t, 0) q_S U(t, 0) \quad (8)$$

A similar equation holds for p_H :

$$p_H(t) = U^\dagger(t, 0) p_S U(t, 0) \quad (9)$$

The Heisenberg and Schrödinger operators are equal at $t = 0$:

$$q_H(0) = q_S \quad (10)$$

$$p_H(0) = p_S \quad (11)$$

We can follow a similar procedure to convert any operator from the Schrödinger to the Heisenberg picture, as Coleman does in eqn 7.15. Note that a general operator $A_S(t)$ may have an explicit time dependence (for example, a Hamiltonian $H(p_S, q_S, t)$), so the general rule is

$$A_H(t) = U^\dagger(t, 0) A_S(t) U(t, 0) \quad (12)$$

$$= U(0, t) A_S(t) U^\dagger(0, t) \quad (13)$$

where the last line follows from

$$U(t', t) = U^{-1}(t, t') = U^\dagger(t, t') \quad (14)$$

PINGBACKS

Pingback: Interaction picture

Pingback: Interaction picture equations of motion