

KLEIN-GORDON EQUATION FROM LAGRANGIAN

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The Klein-Gordon equation is a relativistic description of a spinless massive free particle. The given Lagrangian is

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 \quad (1)$$

Using the Euler-Lagrange equations we have

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2 \phi \quad (2)$$

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} = \partial_\mu \partial^\mu \phi \quad (3)$$

Combining these gives

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \quad (4)$$

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0 \quad (5)$$

which is the Klein-Gordon equation.

The momentum is

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \quad (6)$$

$$= \partial_0 \phi \quad (7)$$

$$= \dot{\phi} \quad (8)$$

The Hamiltonian density is

$$\mathcal{H} = \pi \dot{\phi} - \mathcal{L} \quad (9)$$

$$= \dot{\phi}^2 - \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 \quad (10)$$

$$= \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 \quad (11)$$

This is usually interpreted as the sum of a kinetic energy term $\frac{1}{2} \dot{\phi}^2$ and a potential energy term $\frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2$.

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