

LAGRANGIAN EXAMPLE - INFINITE POLYNOMIAL IN THE SCALAR FIELD

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An example Lagrangian is given by

$$\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 - \sum_{n=1}^{\infty} \lambda_n \phi^{2n+2} \quad (1)$$

I don't know if this Lagrangian describes an actual physical system.

Using the Euler-Lagrange equations to get the equation of motion we must find

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \quad (2)$$

We have

$$-\frac{\partial \mathcal{L}}{\partial \phi} = m^2 \phi + \sum_{n=1}^{\infty} (2n+2) \lambda_n \phi^{2n+1} \quad (3)$$

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} = \partial_\mu \partial^\mu \phi \quad (4)$$

$$= \partial^2 \phi \quad (5)$$

Combining these two derivatives gives the equation of motion as

$$(\partial^2 + m^2) \phi + \sum_{n=1}^{\infty} (2n+2) \lambda_n \phi^{2n+1} = 0 \quad (6)$$

As this equation is nonlinear in the field ϕ , it represents a theory with interactions.