

LAGRANGIAN FOR A RELATIVISTIC FREE PARTICLE

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Given the Lagrangian for a relativistic free particle as

$$L = -\frac{mc^2}{\gamma} = -\sqrt{1 - \frac{v^2}{c^2}} mc^2 \quad (1)$$

we can calculate a few quantities in the non-relativistic limit $v \ll c$. In that case

$$\sqrt{1 - \frac{v^2}{c^2}} \approx 1 - \frac{v^2}{2c^2} \quad (2)$$

so we have

$$L \rightarrow -mc^2 + \frac{1}{2}mv^2 \quad (3)$$

The momentum is defined as

$$p = \frac{\partial L}{\partial \dot{q}} \quad (4)$$

where in this case, $\dot{q} = v$, so we have

$$p \rightarrow mv \quad (5)$$

The Hamiltonian itself is

$$H = p\dot{q} - L \quad (6)$$

$$= mv^2 + mc^2 - \frac{1}{2}mv^2 \quad (7)$$

$$= mc^2 + \frac{1}{2}mv^2 \quad (8)$$

which is the total energy (rest energy mc^2 + kinetic energy $\frac{1}{2}mv^2$).

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