MASS RENORMALIZATION

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In Chapter 10, Coleman begins his analysis of the meson-nucleon model, with the interaction hamiltonian given by

$$\mathscr{H}_I = g\psi^*\psi\phi f(t) \tag{1}$$

Here, the fields ψ and ψ^* are complex fields that represent a nucleon-like particle, and are given by

$$\psi(x) = \int \frac{d^3\boldsymbol{p}}{\sqrt{(2\pi)^3}\sqrt{2\omega_{\boldsymbol{p}}}}\left(b_{\boldsymbol{p}}e^{-ip\cdot x} + c^{\dagger}_{\boldsymbol{p}}e^{ip\cdot x}\right) \tag{2}$$

$$\psi^*(x) = \int \frac{d^3\boldsymbol{p}}{\sqrt{(2\pi)^3}\sqrt{2\omega_{\boldsymbol{p}}}}\left(b^{\dagger}_{\boldsymbol{p}}e^{ip\cdot x} + c_{\boldsymbol{p}}e^{-ip\cdot x}\right) \tag{3}$$

The operators $b_{\boldsymbol{p}}$ and $b^{\dagger}_{\boldsymbol{p}}$ are the annihilation and creation operators for a nucleon, and $c_{\boldsymbol{p}}$ and $c^{\dagger}_{\boldsymbol{p}}$ are the corresponding operators for an antinucleon. Thus ψ can annihilate a nucleon or create an antinucleon, and vice versa for ψ^*.

The field ϕ represents a meson, and is given by

$$\phi(x) = \int \frac{d^3\boldsymbol{p}}{\sqrt{(2\pi)^3}\sqrt{2\omega_{\boldsymbol{p}}}}\left[a_{\boldsymbol{p}}e^{-ip\cdot x} + a^{\dagger}_{\boldsymbol{p}}e^{ip\cdot x}\right] \tag{4}$$

This is a real field, since $\phi^* = \phi$.

The function $f(t)$ in 1 is an 'adiabatic' function which slowly turns the interaction on and then, later, off again.

The Lagrangian for this system is given by

$$\mathscr{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}\mu^2\phi^2 + \partial^\mu\psi^*\partial_\mu\psi - m^2\psi^*\psi - g\psi^*\psi\phi f(t) \tag{5}$$

The first two terms are the free Lagrangian for the meson, the third and fourth terms are the free Lagrangian for the nucleon, and the last term is the interaction term, which appears in the Hamiltonian as $\mathscr{H}_I$ in 1. The mass of the meson is μ (not the μ in a superscript or subscript!) and the mass of the nucleon is m.

Coleman gives a lengthy discussion of the idea of mass renormalization by analogy with a couple of cases from classical physics. If we consider a ping pong ball submerged in water and wish to describe its motion as it floats to the surface, we find that the classical Newtonian equations give the wrong answer if we take the mass of the ping pong ball to be just its mass as measured at rest on dry land. The effective mass of the ball depends, instead, on the density of the liquid in which it is immersed and also on the ball's volume.

Another example is from classical electromagnetism, where the mass of an electron or other charged particle must take into account the mass (really, the energy) of the associated electric field. When moving a charged particle, we must also move the field attached to it, which increases its effective mass.

Reasoning by analogy, Coleman then says that the masses of the meson and nucleon in our model will not be equal to the corresponding masses in the absence of the interaction 1. That is, if the 'bare' mass of the meson in the absence of the interaction is μ_0 and of the nucleon is m_0, then

$$\begin{aligned} \mu_0 &\neq \mu \\ m_0 &\neq m \end{aligned} \tag{6}$$

The masses μ and m are called the *physical masses*, and are what appear in the Lagrangian 5.

We now turn to the calculation of the S matrix. As we argued earlier, the vacuum-to-vacuum matrix element should be 1:

$$\langle 0|S|0\rangle = 1 \tag{7}$$

We ensured this to be the case in the one-field model by introducing a counterterm a which effectively cancels out the change in phase that results from turning the interaction on and off again.

The same argument applies here, so we again introduce a counterterm a into the Lagrangian. In this model, we also need to introduce a couple of other counterterms to deal with the differences in the particles' masses. The idea goes something like this.

Suppose the universe is empty except for a single meson with momentum $\boldsymbol{q}$. If that is the only thing in the universe, then there is nothing for it to interact with, so even if we turn on the interaction term (by increasing $f(t)$ from zero), the meson should just continue along its original path with the same momentum. That is, the S matrix should be 1 in this case as well, so we should have

$$\left\langle \boldsymbol{q}' \left| S \right| \boldsymbol{q} \right\rangle = \left\langle \boldsymbol{q}' \left| \boldsymbol{q} \right. \right\rangle = \delta^{(3)} \left(\boldsymbol{q}' - \boldsymbol{q} \right) \tag{8}$$

The argument appears to be that, in fact, what will happen when we apply the interaction 1 is that meson's mass will change because of the presence of the interaction, so we will get μ_0 morphing into the physical mass μ. However, as the meson is the only thing in the universe, we want its mass to stay at the value of μ_0. Looking at the Lagrangian 5, the term involving the meson's mass is $-\frac{1}{2}\mu^2\phi^2$, so we introduce a counterterm $\frac{1}{2}b\phi^2$ into the interaction to force the mass to remain at μ_0.

The same argument applies to the nucleon, so looking at 5 and observing that the nucelon's mass appears in the term $-m^2\psi^*\psi$, we introduce a counterterm $c\psi^*\psi$. Combining these with the a counterterm, we get the modified Lagrangian

$$\begin{aligned}\mathscr{L} = \frac{1}{2}\left(\partial_\mu\phi\right)^2 - \frac{1}{2}\mu^2\phi^2 + \partial^\mu\psi^*\partial_\mu\psi - m^2\psi^*\psi + \\ \left[-g\psi^*\psi\phi + a + \frac{1}{2}b\phi^2 + c\psi^*\psi\right] f\left(t\right)\end{aligned} \tag{9}$$

Notice that all the counterterms appear multiplied by $f\left(t\right)$, since if the interaction is turned off (that is, $f = 0$), μ and m revert back to μ_0 and m_0, so there's no need for the counterterms in this case.

The counterterms are related to the bare masses by

$$\begin{aligned}\mu_0^2 &= \mu^2 - b \\ m_0^2 &= m^2 - c\end{aligned} \tag{10}$$