MAXIMUM SPACE-TIME PATH FOR FREE PARTICLE

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The Lagrangian for a relativistic free particle is

$$L = -\frac{mc^2}{\gamma} \tag{1}$$

The action integral is therefore

$$S = \int_{\tau_1}^{\tau_2} L\gamma \, d\tau \tag{2}$$

where τ is the particle's proper time, so that the time is $t = \gamma\tau$. The invariant interval ds between two spacetime points is

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \tag{3}$$

In the particle's rest frame where $t = \tau$, the action integral is therefore

$$S = -mc^2 \int_{\tau_1}^{\tau_2} d\tau \tag{4}$$

$$= -mc \int_{\tau_1}^{\tau_2} ds \tag{5}$$

where the particle travels between events a and b, located at proper times τ_1 and τ_2.

The principle of least action requires that this integral be a minimum, which in turn requires that $\int_{\tau_1}^{\tau_2} ds$ be a maximum (because of the minus sign in 5). Assuming the two points a and b are within the light cone, that is, a and b are separated by a timelike interval, which is the only possibility for a single massive particle, then $ds^2 > 0$ and ds is real, so maximizing ds^2 means minimizing $dx^2 + dy^2 + dz^2$. Since the space components of spacetime are Euclidean, the minimum length of a path linking two points in space is a straight line, which is therefore the path followed by a free particle.

Another way of thinking about it is that if we followed some non-straight path, then every deviation from the straight line introduces some extra non-zero contributions from $dx^2 + dy^2 + dz^2$, thus increasing it and thus reducing ds^2.