

MOMENTUM AND ENERGY FOR A FREE PARTICLE IN AN ELECTROMAGNETIC FIELD

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Post date: 29 Jul 2023.

The Lagrangian for a free particle in an electromagnetic field is

$$L = -\frac{mc^2}{\gamma} + q\mathbf{A}(x) \cdot \mathbf{v} - qV(x) \quad (1)$$

In the non-relativistic limit, we've seen that for a totally free particle (no EM field), the Lagrangian becomes

$$L = -\frac{mc^2}{\gamma} = -\sqrt{1 - \frac{v^2}{c^2}} mc^2 \rightarrow -mc^2 + \frac{1}{2}mv^2 \quad (2)$$

In this limit, the Lagrangian 1 thus becomes

$$L \rightarrow -mc^2 + \frac{1}{2}mv^2 + q\mathbf{A}(x) \cdot \mathbf{v} - qV(x) \quad (3)$$

The canonical momentum is

$$p_i = \frac{\partial L}{\partial v_i} \quad (4)$$

$$= mv_i + qA_i \quad (5)$$

The energy is given by the Hamiltonian

$$E = p_i v_i - L \quad (6)$$

$$= mv_i v_i + qA_i v_i + mc^2 - \frac{1}{2}mv_i v_i - qA_i v_i + qV \quad (7)$$

$$= mc^2 + \frac{1}{2}mv_i v_i + qV \quad (8)$$

We can eliminate the velocity in favour of the momentum by using 5:

$$v_i = \frac{1}{m}(p_i - qA_i) \quad (9)$$

so we get

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$$E = mc^2 + \frac{1}{2m} (p_i - qA_i)(p_i - qA_i) + qV \quad (10)$$

In vector notation, this is

$$E = mc^2 + \frac{1}{2m} (\mathbf{p} - q\mathbf{A})^2 + qV \quad (11)$$