

SCHRÖDINGER PICTURE

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The Schrödinger picture in quantum mechanics is the one in which all the time dependence is placed in the quantum states $|\psi(t)\rangle$ and not in the operators. The fundamental operators, position q and momentum p are not time-dependent, so we can write them as q_S and p_S (subscript S for Schrödinger) without any time argument. The states obey the standard Schrödinger equation

$$i \frac{d}{dt} |\psi(t)\rangle = H(p_S, q_S, t) |\psi(t)\rangle \quad (1)$$

The Hamiltonian may have an explicit time dependence, but if it doesn't, it doesn't have any indirect time dependence either since the operators p_S and q_S are time-independent.

The evolution of a Schrödinger state is obtained from a time evolution operator $U(t, t')$ (also known as the propagator) according to

$$|\psi(t)\rangle = U(t, t') |\psi(t')\rangle \quad (2)$$

This operator takes a state at time t' and evolves it to another time t (t could be earlier than, later than or equal to t'). U is a linear operator because the Schrödinger equation is linear, so the state $|\psi(t)\rangle$ must obey the Schrödinger equation at all times. Because the Schrödinger equation conserves probability, we must have, assuming the state is normalized:

$$\langle \psi(t) | \psi(t) \rangle = 1 \quad (3)$$

for all times, so we must have

$$\langle U(t, t') \psi(t') | U(t, t') \psi(t') \rangle = \langle \psi(t') U^\dagger(t, t') | U(t, t') \psi(t') \rangle \quad (4)$$

$$= \langle \psi(t') | \psi(t') \rangle \quad (5)$$

The equality of these two lines is a requirement, not a derivation, so we can deduce that we must have

$$U^\dagger(t, t') = U^{-1}(t, t') \quad (6)$$

which means the operator U is unitary. There is also a composition rule:

$$U(t, t'') = U(t, t') U(t', t'') \quad (7)$$

Combining this with 6 we have

$$U(t', t) = U^{-1}(t, t') \quad (8)$$

A differential equation for U can be obtained by differentiating 2 and using 1. First we differentiate 2

$$i \frac{d}{dt} |\psi(t)\rangle = i \frac{\partial}{\partial t} U(t, t') |\psi(t')\rangle \quad (9)$$

Next we have from 1

$$i \frac{d}{dt} |\psi(t)\rangle = H(p_S, q_S, t) |\psi(t)\rangle \quad (10)$$

$$= H(p_S, q_S, t) U(t, t') |\psi(t')\rangle \quad (11)$$

Comparing 9 and 11 we have

$$i \frac{\partial}{\partial t} U(t, t') = H(p_S, q_S, t) U(t, t') \quad (12)$$

with the initial condition

$$U(t', t') = 1 \quad (13)$$

This is just saying that if we evolve a state through zero time, nothing changes. If H doesn't depend explicitly on t , a formal solution of 12 is

$$U(t, t') = e^{-iH(p_S, q_S)(t-t')} \quad (14)$$

The RHS assumes that we can expand the exponential as a power series, so we get a series involving powers of the hamiltonian. If H *does* depend explicitly on time, then the integration of 12 can be a lot more complicated.

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