SCHRÖDINGER PICTURE

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The Schrödinger picture in quantum mechanics is the one in which all the time dependence is placed in the quantum states $|\psi(t)\rangle$ and not in the operators. The fundamental operators, position q and momentum p are not time-dependent, so we can write them as q_S and p_S (subscript S for Schrödinger) without any time argument. The states obey the standard Schrödinger equation

$$i\frac{d}{dt}|\psi(t)\rangle = H(p_S, q_S, t)|\psi(t)\rangle \tag{1}$$

The Hamiltonian may have an explicit time dependence, but if it doesn't, it doesn't have any indirect time dependence either since the operators p_S and q_S are time-independent.

The evolution of a Schrödinger state is obtained from a time evolution operator $U(t, t')$ (also known as the propagator) according to

$$|\psi(t)\rangle = U(t, t')|\psi(t')\rangle \tag{2}$$

This operator take a state at time t' and evolves it to another time t (t could be earlier than, later than or equal to t'). U is a linear operator because the Schrödinger equation is linear, so the state $|\psi(t)\rangle$ must obey the Schrödinger equation at all times. Because the Schrödinger equation conserves probability, we must have, assuming the state is normalized:

$$\langle\psi(t)|\psi(t)\rangle = 1 \tag{3}$$

for all times, so we must have

$$\begin{aligned}
\left\langle U(t,t')\psi(t')\middle|U(t,t')\psi(t')\right\rangle &= \left\langle \psi(t')U^{\dagger}(t,t')\middle|U(t,t')\psi(t')\right\rangle & (4)\\
&= \left\langle\psi(t')\middle|\psi(t')\right\rangle & (5)
\end{aligned}$$

The equality of these two lines is a requirement, not a derivation, so we can deduce that we must have

$$U^{\dagger}(t,t') = U^{-1}(t,t') \tag{6}$$

which means the operator U is unitary. There is also a composition rule:

$$U\left(t,t''\right)=U\left(t,t'\right)U\left(t',t''\right) \tag{7}$$

Combining this with 6 we have

$$U\left(t',t\right)=U^{-1}\left(t,t'\right) \tag{8}$$

A differential equation for U can be obtained by differentiating 2 and using 1. First we differentiate 2

$$i\frac{d}{dt}\left|\psi\left(t\right)\right\rangle =i\frac{\partial}{\partial t}U\left(t,t'\right)\left|\psi\left(t'\right)\right\rangle \tag{9}$$

Next we have from 1

$$\begin{aligned} i\frac{d}{dt}\left|\psi\left(t\right)\right\rangle &= H\left(p_{S},q_{S},t\right)\left|\psi\left(t\right)\right\rangle && (10)\\ &= H\left(p_{S},q_{S},t\right)U\left(t,t'\right)\left|\psi\left(t'\right)\right\rangle && (11)\end{aligned}$$

Comparing 9 and 11 we have

$$i\frac{\partial}{\partial t}U\left(t,t'\right)=H\left(p_{S},q_{S},t\right)U\left(t,t'\right) \tag{12}$$

with the initial condition

$$U\left(t',t'\right)=1 \tag{13}$$

This is just saying that if we evolve a state through zero time, nothing changes. If H doesn't depend explicitly on t, a formal solution of 12 is

$$U\left(t,t'\right)=e^{-iH\left(p_{S},q_{S}\right)\left(t-t'\right)} \tag{14}$$

The RHS assumes that we can expand the exponential as a power series, so we get a series involving powers of the hamiltonian. If H *does* depend explicitly on time, then the integration of 12 can be a lot more complicated.

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