

TIME EVOLUTION OF SPIN IN A MAGNETIC FIELD

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The Heisenberg equation of motion for an operator Q is

$$\frac{dQ}{dt} = -\frac{i}{\hbar} [Q, H] \quad (1)$$

where H is the Hamiltonian.

Although we've looked at a spin- $\frac{1}{2}$ particle in a magnetic field before, here we'll do a simplified treatment. The Hamiltonian for the interaction of a particle with a constant magnetic field along the y direction can be written as

$$H = \omega S_y \quad (2)$$

where ω is a constant proportional to the magnetic field. The time evolution of the other two spin components can then be found from 1. We have

$$\frac{dS_z}{dt} = -\frac{i}{\hbar} [S_z, \omega S_y] \quad (3)$$

$$= -\frac{i\omega}{\hbar} [S_z, S_y] \quad (4)$$

$$= \frac{i\omega}{\hbar} [S_y, S_z] \quad (5)$$

$$= \frac{i\omega}{\hbar} (i\hbar S_x) \quad (6)$$

$$= -\omega S_x \quad (7)$$

where to get the fourth line, we've used the angular momentum commutator

$$[S_y, S_z] = i\hbar S_x \quad (8)$$

Similarly, for S_x we have

$$\frac{dS_x}{dt} = -\frac{i}{\hbar} [S_x, \omega S_y] \quad (9)$$

$$= -\frac{i\omega}{\hbar} [S_x, S_y] \quad (10)$$

$$= \omega S_z \quad (11)$$

Thus the components of spin perpendicular to the magnetic field change proportionally to each other. This is the precession of the spin about the magnetic field.