COULOMB'S LAW

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Sec. 2.1.3-2.1.4.

Electrostatics is the study of electric fields produced by collections of stationary charges. The basic problem that is solved in electrostatics is: given a collection or continuous distribution of charges, what is the net force on a test charge Q placed at a given position?

Experimentally, there are two principles or laws in electrostatics, both arising out of experiment (so they aren't derived from anything more fundamental). The first is Coulomb's law. Suppose we have one charge q at position $\mathbf{r}'$ and a test charge Q at position $\mathbf{r}$. Coulomb's law states that the electric force between the charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The force is directed along a line parallel to the distance between the two charges. As an equation, this comes out to

$$\mathbf{F}=\frac{1}{4\pi\epsilon_0}\frac{qQ}{|\mathbf{r}-\mathbf{r}'|^2}\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \tag{1}$$

We've written the law in this way so that the last factor on the right produces a unit vector in the direction $\mathbf{r}-\mathbf{r}'$, which is a vector pointing from q to Q.

The constant ϵ_0 is known as the *permittivity of free space* and in SI units has the value

$$\epsilon_0 = 8.85\times10^{-12}C^2/N\cdot m^2 \tag{2}$$

where charge is measured in coulombs (C), force in newtons (N) and distance in metres (m). The constant in Coulomb's law thus comes out to

$$\frac{1}{4\pi\epsilon_0}=8.99\times10^9 N\cdot m^2/C^2 \tag{3}$$

The second principle in electrostatics is the *principle of superposition*. This states that in a system with two or more point charges, or with a continuous distribution of charge (or a combination of the two), the total net force on the test charge Q is simply the vector sum of the forces due to each charge in the discrete case, and the integral of the forces in the continuous case.

This is a non-trivial assumption so it shouldn't be glossed over. It states that the force between two charges is independent of any other charges that may be present. The same assumption is true in Newton's theory of gravity.

Mathematically, the principle of superposition means that that total force on Q from a collection of point charges is

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 Q}{|\mathbf{r}-\mathbf{r}_1'|^2}\frac{(\mathbf{r}-\mathbf{r}_1')}{|\mathbf{r}-\mathbf{r}_1'|} + \frac{1}{4\pi\epsilon_0}\frac{q_2 Q}{|\mathbf{r}-\mathbf{r}_2'|^2}\frac{(\mathbf{r}-\mathbf{r}_2')}{|\mathbf{r}-\mathbf{r}_2'|} + \ldots \tag{4}$$

$$= \frac{Q}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{|\mathbf{r}-\mathbf{r}_i'|^2}\frac{(\mathbf{r}-\mathbf{r}_i')}{|\mathbf{r}-\mathbf{r}_i'|} \tag{5}$$

Ideally, we would like a way of describing the effect of a collection of charges without referring to a particular test charge, and from the above equation we can see a way of doing this. If we define $\mathbf{F} = Q\mathbf{E}$, we get

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{|\mathbf{r}-\mathbf{r}_i'|^2}\frac{(\mathbf{r}-\mathbf{r}_i')}{|\mathbf{r}-\mathbf{r}_i'|} \tag{6}$$

where $\mathbf{E}$ is called the *electric field*. We can see that it depends on the point $\mathbf{r}$ where the test charge would be placed, but does not depend on the test charge itself.

For a continuous charge distribution, the corresponding definition of $\mathbf{E}$ is a volume integral:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}dq \tag{7}$$

where $\mathbf{r}'$ is the location of the infinitesimal charge dq. The integral extends over all spatial locations containing charge.

The equations 6 and 7 are all we need to know in order to solve problems in electrostatics. As generations of physics students know, however, actually finding $\mathbf{E}$ in a huge variety of geometric situations can provide endless hours of toil in trying to work out exotic sums and integrals.

On the philosophical side, one can ask what exactly the field $\mathbf{E}$ is, physically (apart from a notational convenience in the working out of a force). Does an electric field exist in any independent sense? It's a bit like asking if there is any sound if a tree falls in the forest and there is nobody there to hear it. The only way an electric field can be detected is by placing a test charge at some location and measuring the force on it. However, since a test charge would feel no force in the absence of an electric field, it seems legitimate to regard the electric field as something real.

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