ELECTROSTATICS - POINT CHARGES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Sec 2.1.3.

In our first look at electrostatics, we saw that a collection of charges creates an electric field **E** according to the formula

$$\mathbf{E}(\mathbf{r})=\frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{|\mathbf{r}-\mathbf{r}_i'|^2}\frac{(\mathbf{r}-\mathbf{r}_i')}{|\mathbf{r}-\mathbf{r}_i'|}\tag{1}$$

where $\mathbf{r}_i'$ is location of charge q_i. We can work out a couple of examples to show how the field is calculated.

First, suppose we have a number of identical charges q with each charge placed at the vertex of a regular polygon. What would be the electric field at the centre of the polygon?

Although it is possible to work this out using vectors that point from the centre to each vertex, we can observe that by symmetry, the field at the centre must be zero. If it were not zero, it would have to point towards some part of the polygon, but since the polygon is completely symmetric, there is no way to define a preferred direction.

If one of the charges is removed, then again by symmetry we know that there is now an unbalanced field in the direction of the removed charge. The magnitude of the field is $\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ where r here is the distance from the centre to the removed charge (that is, the distance from the centre to any vertex).

As a second example, suppose that two identical charges q are placed on the x-axis at locations $-d/2$ and $d/2$. What is the electric field at a point P on the z axis?

The electric field is

$$\mathbf{E}=\frac{1}{4\pi\epsilon_0}\frac{q}{(z^2+d^2/4)^{3/2}}\left[-\frac{d}{2},0,z\right]+\frac{1}{4\pi\epsilon_0}\frac{q}{(z^2+d^2/4)^{3/2}}\left[\frac{d}{2},0,z\right]\tag{2}$$

$$=\frac{1}{4\pi\epsilon_0}\frac{2q}{(z^2+d^2/4)^{3/2}}[0,0,z]\tag{3}$$

Note that the components of the field in the x direction cancel each other and we are left with a field parallel to the z axis.

For $z\gg d$, we get

$$\mathbf{E} \to \frac{1}{4\pi\epsilon_0}\frac{2q}{z^2}\hat{k} \tag{4}$$

In other words, at a large distance from the two charges, they appear as a single charge of $2q$, as expected.

With opposite charges, we get

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{-q}{(z^2+d^2/4)^{3/2}}\left[-\frac{d}{2},0,z\right] + \frac{1}{4\pi\epsilon_0}\frac{q}{(z^2+d^2/4)^{3/2}}\left[\frac{d}{2},0,z\right] \tag{5}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{(z^2+d^2/4)^{3/2}}[d,0,0] \tag{6}$$

For $z \gg d$, we get

$$\mathbf{E} \to \frac{1}{4\pi\epsilon_0}\frac{qd}{z^3}\hat{i} \tag{7}$$

This is a special case of the electric dipole formula. A dipole is two charges with equal but opposite charge, and the dipole axis is the line connecting the two charges. The electric field on a line perpendicular to the dipole axis is parallel to the axis and is inverse cube with distance from the dipole. A more general formula would give the field due to the dipole at all points in space, but that's bit more complicated than we want to get into here.

PINGBACKS

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