ELECTRIC POTENTIAL FROM CHARGES - EXAMPLES 2

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problems 2.27 - 2.28.

Here are a couple more examples of calculating the potential from the charge distribution.

Example 1. Given a uniformly charged solid sphere of radius R and total charge q, find the potential from the formula

$$V(\mathbf{r})=\frac{1}{4\pi\epsilon_0}\int\frac{1}{|\mathbf{r}-\mathbf{r}'|}\rho(\mathbf{r}')d^3\mathbf{r}' \tag{1}$$

In this case, the charge density is constant, and can be expressed in terms of q by

$$\rho=\frac{3q}{4\pi R^3} \tag{2}$$

Using the cosine law, we have

$$|\mathbf{r}-\mathbf{r}'|=\sqrt{r^2+r'^2-2rr'\cos\theta} \tag{3}$$

so in spherical coordinates, we get

$$V(\mathbf{r})=\frac{3q}{16\pi^2\epsilon_0 R^3}\int_0^{2\pi}\int_0^R\int_0^\pi\frac{r'^2\sin\theta d\theta dr'd\phi}{\sqrt{r^2+r'^2-2rr'\cos\theta}} \tag{4}$$

$$=\frac{q}{8\pi\epsilon_0 R}\left(3-\frac{r^2}{R^2}\right) \tag{5}$$

which agrees with the result in Example 1 in an earlier post.

Example 2. Given a uniformly charged solid cylinder of length L, radius R and charge density ρ with its axis along the z axis and centre at the origin, find the potential at location z_V on the z axis, where we're assuming that $z_V>L/2$.

We can use the result of Example 3 in an earlier post. The potential of a charged disk is

$$V=\frac{\sigma}{2\epsilon_0}\left[\sqrt{z^2+R^2}-z\right] \tag{6}$$

where z is the distance above the centre of the disk. In the case of the cylinder, we can slice the cylinder into a number of disks, each of thickness dz. For a slice at height z the distance from the centre of the slice to location z_V is then $z_V - z$, so the potential of the entire cylinder will be

$$V = \frac{\rho}{2\epsilon_0}\int_{-L/2}^{L/2}\left[\sqrt{(z_V - z)^2 + R^2} - z_V + z\right]dz \tag{7}$$

This is an unpleasant integral, but can be done with software. Maple gives (after condensing a few of the terms):

$$\frac{2\epsilon_0}{\rho}V = \sqrt{(2z_V + L)^2 + (2R)^2}\left(\frac{L}{8} + \frac{z_V}{4}\right) + \sqrt{(2z_V - L)^2 + (2R)^2}\left(\frac{L}{8} - \frac{z_V}{4}\right) \tag{8}$$

$$+\frac{R^2}{2}\ln\left[\frac{\sqrt{(2z_V - L)^2 + (2R)^2} - 2z_V + L}{\sqrt{(2z_V + L)^2 + (2R)^2} - 2z_V - L}\right] - z_V L \tag{9}$$

Since the system has symmetry about the z axis, we can calculate the electric field along the axis by taking the gradient. After using Maple and doing some simplification we get (remember that we take the derivative w.r.t. z_V in the gradient):

$$\mathbf{E} = -\nabla V \tag{10}$$

$$= \frac{\rho}{2\epsilon_0}\left[L - \frac{1}{2}\sqrt{(2z_V + L)^2 + (2R)^2} + \frac{1}{2}\sqrt{(2z_V - L)^2 + (2R)^2}\right]\hat{\mathbf{z}} \tag{11}$$

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