

## ELECTRIC POTENTIAL FROM CHARGES - EXAMPLES 2

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the auxiliary blog.

Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problems 2.27 - 2.28.

Here are a couple more examples of calculating the potential from the charge distribution.

**Example 1.** Given a uniformly charged solid sphere of radius  $R$  and total charge  $q$ , find the potential from the formula

$$(1) \quad V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{1}{|\mathbf{r} - \mathbf{r}'|} \rho(\mathbf{r}') d^3\mathbf{r}'$$

In this case, the charge density is constant, and can be expressed in terms of  $q$  by

$$(2) \quad \rho = \frac{3q}{4\pi R^3}$$

Using the cosine law, we have

$$(3) \quad |\mathbf{r} - \mathbf{r}'| = \sqrt{r^2 + r'^2 - 2rr' \cos \theta}$$

so in spherical coordinates, we get

$$(4) \quad V(\mathbf{r}) = \frac{3q}{16\pi^2\epsilon_0 R^3} \int_0^{2\pi} \int_0^R \int_0^\pi \frac{r'^2 \sin \theta d\theta dr' d\phi}{\sqrt{r^2 + r'^2 - 2rr' \cos \theta}}$$

$$(5) \quad = \frac{q}{8\pi\epsilon_0 R} \left( 3 - \frac{r^2}{R^2} \right)$$

which agrees with the result in Example 1 in an earlier post.

**Example 2.** Given a uniformly charged solid cylinder of length  $L$ , radius  $R$  and charge density  $\rho$  with its axis along the  $z$  axis and centre at the origin, find the potential at location  $z_V$  on the  $z$  axis, where we're assuming that  $z_V > L/2$ .

We can use the result of Example 3 in an earlier post. The potential of a charged disk is

$$(6) \quad V = \frac{\sigma}{2\epsilon_0} \left[ \sqrt{z^2 + R^2} - z \right]$$

where  $z$  is the distance above the centre of the disk. In the case of the cylinder, we can slice the cylinder into a number of disks, each of thickness  $dz$ . For a slice at height  $z$  the distance from the centre of the slice to location  $z_V$  is then  $z_V - z$ , so the potential of the entire cylinder will be

$$(7) \quad V = \frac{\rho}{2\epsilon_0} \int_{-L/2}^{L/2} \left[ \sqrt{(z_V - z)^2 + R^2} - z_V + z \right] dz$$

This is an unpleasant integral, but can be done with software. Maple gives (after condensing a few of the terms):

$$(8) \quad \frac{2\epsilon_0}{\rho} V = \sqrt{(2z_V + L)^2 + (2R)^2} \left( \frac{L}{8} + \frac{z_V}{4} \right) + \sqrt{(2z_V - L)^2 + (2R)^2} \left( \frac{L}{8} - \frac{z_V}{4} \right)$$

$$(9) \quad + \frac{R^2}{2} \ln \left[ \frac{\sqrt{(2z_V - L)^2 + (2R)^2} - 2z_V + L}{\sqrt{(2z_V + L)^2 + (2R)^2} - 2z_V - L} \right] - z_V L$$

Since the system has symmetry about the  $z$  axis, we can calculate the electric field along the axis by taking the gradient. After using Maple and doing some simplification we get (remember that we take the derivative w.r.t.  $z_V$  in the gradient):

$$(10) \quad \mathbf{E} = -\nabla V$$

$$(11) \quad = \frac{\rho}{2\epsilon_0} \left[ L - \frac{1}{2} \sqrt{(2z_V + L)^2 + (2R)^2} + \frac{1}{2} \sqrt{(2z_V - L)^2 + (2R)^2} \right] \hat{\mathbf{z}}$$

#### PINGBACKS

Pingback: [Work and energy - continuous charge](#)

Pingback: [Laplace's equation - average values of solutions](#)

Pingback: [Laplace's equation - charged disk](#)