

ELECTROSTATIC BOUNDARY CONDITIONS - EXAMPLES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 2.30.

We've seen that the electric field has a discontinuity of σ/ϵ_0 when we cross a surface charge of density σ , but that the potential is continuous. Here we offer some examples that verify this condition.

Example 1. From Example 1 in a previous post, we know that the electric field for an infinite plane is

$$\mathbf{E} = \frac{\sigma}{2\epsilon_0} \hat{\mathbf{n}} \quad (1)$$

where σ is the surface charge density and $\hat{\mathbf{n}}$ is a unit vector normal to the plane, pointing away from the plane on both sides. The difference in field as we cross the plane is therefore σ/ϵ_0 .

Example 2. If we have two infinite, parallel planes of charge, with one having a charge density of $+\sigma$ and the other of $-\sigma$, then by the principle of superposition, the field will be zero everywhere except between the planes, where it will be σ/ϵ_0 pointing from the positive plane to the negative plane. Crossing either plane thus gives rise to a discontinuity in the field of σ/ϵ_0 .

Example 3. For a spherical shell of charge, from Example 1 in a previous post, we know that the field is $E = 0$ inside the shell and points radially outward with a magnitude of

$$E = \frac{R^2 \sigma}{\epsilon_0 r^2} \quad (2)$$

where R is the radius of the sphere and r is the distance from the centre. At the boundary, $r = R$ and $E = \sigma/\epsilon_0$ so the discontinuity across the shell is again σ/ϵ_0 .

Example 4. Consider an infinitely long, hollow, cylindrical tube with surface charge density σ . By symmetry, the field points radially outwards, so we can use Gauss's law to find it. If the radius of the tube is R and we consider a length L then if we consider a Gaussian cylinder enclosing this section, the total enclosed charge is $2\pi RL\sigma$. If the Gaussian cylinder has radius r its area (minus the end caps, which don't contribute) is $2\pi rL$ so

$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{q}{\epsilon_0} \quad (3)$$

$$2\pi rLE = \frac{2\pi RL\sigma}{\epsilon_0} \quad (4)$$

$$E = \frac{R\sigma}{r\epsilon_0} \quad (5)$$

At the boundary $r = R$ so the field outside is σ/ϵ_0 . Inside, since there is no enclosed charge, $E = 0$ so again the discontinuity at the boundary is σ/ϵ_0 .

Example 5. We can work out the potential due to a spherical shell of charge from the formula

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{1}{|\mathbf{r} - \mathbf{r}'|} \rho(\mathbf{r}') d\mathbf{r}' \quad (6)$$

If we consider a sphere of radius R , and determine the potential at a point z on the z axis, then

$$|\mathbf{r} - \mathbf{r}'| = \sqrt{z^2 + R^2 - 2zr \cos \theta} \quad (7)$$

where θ is the angle between the z axis and the vector \mathbf{r}' . The integral we need to do in this case is

$$V(z) = \frac{2\pi\sigma}{4\pi\epsilon_0} \int_0^\pi \frac{R^2 \sin \theta d\theta}{\sqrt{z^2 + R^2 - 2zr \cos \theta}} \quad (8)$$

The value of this integral depends on the relative sizes of z and R . If $z < R$ so we are inside the sphere, then

$$V(z < R) = \frac{R\sigma}{\epsilon_0} \quad (9)$$

If $z > R$ so we are outside the sphere, then

$$V(z > R) = \frac{R^2\sigma}{z\epsilon_0} \quad (10)$$

At the boundary, $z = R$ and V is $R\sigma/\epsilon_0$ on both sides, so it is continuous. The gradient is zero inside the shell, while outside it is

$$-\nabla V = \frac{R^2\sigma}{z^2\epsilon_0} \quad (11)$$

so at the boundary $z = R$ and the gradient is σ/ϵ_0 . Thus we get the discontinuity of the field as before.

PINGBACKS

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