ELECTROSTATICS - CONDUCTORS

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Sec 2.5, Problems 2.35 - 2.36.

A conductor is a substance containing charge that is free to move under the influence of an electric field. A perfect conductor contains an unlimited amount of free charge. Many metals come quite close to being perfect conductors.

The microscopic processes that occur within a conductor are the subject of more advanced physics, but for now, we can visualize a conductor as a substance in which one or more electrons per atom are free to move throughout the volume of the conductor. Thus the charge that moves is usually negative charge (since electrons are negative), although we often talk about both positive and negative charge moving within a conductor. The motion of 'positive' charge is accomplished by the motion of negative charge in the opposite direction.

Conductors have several properties that at first glance seem quite strange.

Property 1. $\mathbf{E}=0$ everywhere inside a conductor. The simple explanation for this is that, if $\mathbf{E}\neq 0$ somewhere inside a conductor, then any charge present there would move in response to the field, and rearrange itself until the field was zero. We can see how this works by considering what happens if a conductor is placed in an external electric field. The field will act on the charge inside the conductor, and the charge will move. Eventually this charge will hit a boundary of the conductor and be forced to stop. This motion of the charge will generate its own electric field, which will continue to adjust itself until the net field (external + induced) inside the conductor is zero, and the charge stops moving.

Property 2. The electric field at the outside surface of a conductor is normal to the surface. This follows from a similar argument as Property 1. If there were any component of $\mathbf{E}$ parallel to the surface, it would cause charge to move along the surface until such a component was cancelled out.

Property 3. The charge density $\rho=0$ everywhere inside a conductor. This follows from Property 1 and Gauss's law. Since the field is zero everywhere inside, then $\nabla\cdot\mathbf{E}=\rho/\epsilon_0=0$ everywhere inside as well. Note that this does *not* imply that the surface charge density is zero on a conductor. As we saw in Property 1, free charge will be forced to the surface by an external field, so charge tends to pile up there. Property 2 states that any force felt by charge at the surface of a conductor is normal (and outward) at the surface, so this surface charge gets stuck there by the balance of the outward electric field and atomic forces holding the charge inside the conductor.

Property 4. The potential is constant everywhere within a conductor. This follows from the fact that $\mathbf{E}=0$ and $\mathbf{E}=-\nabla V$.

An external electric field, such as that from a point charge or other charge distribution placed nearby, will induce a surface charge distribution on a conductor. For example, if we place a positive point charge $+q$ near a solid metal sphere, the side of the sphere nearest the charge will accumulate some negative charge, with a corresponding build up of positive charge on the far side of the sphere. Thus the point charge and the sphere will attract each other.

There are various mathematical techniques for calculating the induced charge distributions in conductors, but we'll leave most of these till later posts. Here we'll consider the special case of a cavity inside a solid conductor.

Suppose we have a solid conductor (the precise shape doesn't matter at this stage) and it contains a cavity (again, the precise shape doesn't matter - all we're requiring here is that the cavity is totally surrounded by the conductor, so there are no holes leading to the outside world). If a point charge $+q$ is placed inside the cavity, what happens?

This point charge generates a field that must be cancelled by charge redistributing itself inside the conductor. Since the point charge is positive, it will attract a negative charge distribution on the inner wall of the cavity, and this charge will distribute itself in such a way that the electric field in the conductor surrounding the cavity is zero. By applying Gauss's law to a volume enclosing the entire cavity (that is, the wall of the cavity and the point charge within it) we see that the amount of negative charge that is attracted to the cavity's inner surface must be precisely $-q$. Why? Because the electric field on the surface of the Gaussian volume is zero (it's all inside the conductor), so the total enclosed charge must also be zero. (Remember Gauss's law says that $\oint\mathbf{E}\cdot d\mathbf{a}=Q/\epsilon_{0}$ where Q is the total charge enclosed by the surface.)

Since a total of $-q$ has been attracted to the inner surface, an amount of charge $+q$ must therefore distribute itself over the outer surface of the conductor in order to maintain electric neutrality. The combination of the fields from these three sources (point charge in the cavity, induced negative charge on the inner surface, and positive charge on the outer surface) conspire to provide a zero field inside the conductor.

Now let's get a bit more specific and require the conductor to be a solid metal sphere with a cavity (the cavity can still be any shape) inside it. How does the charge on the outer surface distribute itself?

To answer this question fully requires that we prove that the configuration giving rise to the condition that $\mathbf{E}=0$ inside the conductor is unique, but that will have to wait till another post. What we can do is show that there is at least one configuration that does satisfy this requirement (and once we know that the configuration is unique, we know we have the only answer).

The argument goes like this: we suppose that the negative charge that gets induced on the inner surface is capable of neutralizing the effect of the point charge without any help from the positive charge on the outer surface. Is it reasonable to say that? Yes, because we can visualize a situation where the sphere is so large (light years in diameter if you like) that any charge on the outer surface is so far away it has negligible effect on the area around the cavity. If the inner surface and point charge fields cancel, the positive charge will distribute itself over the outer surface uniformly, since we know that the field inside a spherical shell is zero (see Example 1 in this post). In other words, we are claiming that there are two separate charge distributions (one consisting of the point charge and the cavity wall; the other from the outer surface) and that the field inside the conductor is separately zero from each of these two distributions.

We therefore reach the surprising conclusion that, for a conducting sphere, the outer charge distribution does not depend on the shape or location of the cavity; the sphere will have charge $+q$ (equal to the point charge inside the cavity) distributed uniformly over its surface.

We'll now look at a couple of examples of how we can apply this.

Example 1. A conducting sphere of radius R carries a net charge q. This sphere is surrounded by a concentric conducting shell with an inner radius a and outer radius b. The shell has no net charge.

First, the inner sphere must have a surface charge density of $\sigma_R = q/4\pi R^2$ (total charge divided by surface area of the sphere). The charge must be entirely on the surface of the sphere, and is distributed uniformly due to symmetry.

This situation is an instance of a sphere (the concentric shell) containing a cavity, except that the charge inside the cavity is the inner sphere rather than a point charge. However, outside the inner sphere, the field behaves like a point charge of size $+q$, so there is a negative charge of $-q$ induced on the inner surface of the shell. Because of the symmetry, we can now say that this charge is distributed uniformly over the inner surface, so the charge density is $\sigma_a = -q/4\pi a^2$. The charge density on the outer surface is then $\sigma_b = q/4\pi b^2$.

If the potential is zero at infinity, we can find it at each point in the system by integrating.

$$V(b) = -\int_{\infty}^{b} \mathbf{E} \cdot d\mathbf{l} \tag{1}$$

$$= -\int_{\infty}^{b} \frac{q}{4\pi\epsilon_0 r^2} dr \tag{2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{b} \tag{3}$$

Inside the shell, $\mathbf{E} = 0$ so the potential remains constant until we reach the inner surface. Then we go from $r = a$ to $r = R$:

$$V(R) = V(b) - \int_{a}^{R} \frac{q}{4\pi\epsilon_0 r^2} dr \tag{4}$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{b} + \frac{1}{R} - \frac{1}{a} \right] \tag{5}$$

The potential then remains constant inside the inner sphere.

If we then ground the outer surface of the shell so that its potential is $V(b) = 0$, we simply lose the $1/b$ term in the second answer:

$$V_{grounded}(R) = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{R} - \frac{1}{a} \right] \tag{6}$$

Example 2. We have a solid metal sphere of radius R which contains two non-overlapping spherical cavities, one of radius a and the other of radius b. Inside the first cavity is a point charge q_a and in the other is a charge q_b. By the principle of superposition, we can say that the induced surface charge density in cavity a is $\sigma_a = -q_a/4\pi a^2$ and that in cavity b is $\sigma_b = -q_b/4\pi b^2$. The induced charge on the outer surface of the sphere is thus $q_a + q_b$ so the surface density is $\sigma_R = (q_a + q_b)/4\pi R^2$.

The field outside the sphere is therefore just

$$E_{out} = \frac{q_a + q_b}{4\pi\epsilon_0 r^2} \tag{7}$$

where r is measured from the centre of the sphere. $\mathbf{E}$ points radially outward. In other words, the external field is equivalent to that from a combined point charge of $q_a + q_b$.

The field within each cavity is due entirely to the point charge within that cavity, since the field from the charge in the other cavity is sheilded by the intervening conductor. Thus within cavity a

$$E_a = \frac{q_a}{4\pi\epsilon_0 r_a^2} \tag{8}$$

where r_a is measured from the centre of cavity a. Same for cavity b (replace subscript a by b).

Since each point charge is situated within spherical shells of induced charge (one on the inner surface of the cavity and one on the outer surface of the sphere) the field felt from both these shells is zero, so the force is zero.

If we now placed a third charge outside the sphere, what would happen? The induced charges on the inner surfaces of the cavities wouldn't change. The charge distribution on the surface of the sphere *would* change, but it would do so in such a way as to retain a zero field inside the conductor. Thus there is still no force on either of q_a or q_b.

PINGBACKS

Pingback: Electrostatic pressure
Pingback: Capacitance
Pingback: Method of images
Pingback: Method of images - point charge and sphere
Pingback: Green's reciprocity theorem
Pingback: Dielectric examples
Pingback: Dipole between two angled conducting planes
Pingback: Decay time for free charge in a conductor
Pingback: Wave guides: derivation of the wave equation
Pingback: Radiation from a point charge near a conducting plane