CONSERVATION OF ENERGY IN A CAPACITOR

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 2.40.

The electrostatic pressure (force per unit area) on the surface of a conductor is

$$P=\frac{\epsilon_0}{2}E^2 \tag{1}$$

where E is the electric field just outside the conductor ($E=0$ inside a conductor).

Suppose we have a flat parallel plate capacitor with plate area A carrying an amount of charge so that the field between the plates is E. Each plate experiences a pressure as above, so the total force on each plate is

$$F=PA=\frac{\epsilon_0}{2}E^2A \tag{2}$$

If the plates move an infinitesimal distance ϵ towards each other under the influence of this force, the work done is

$$\begin{aligned} W &= PA\epsilon & (3)\\ &= \frac{\epsilon\epsilon_0}{2}AE^2 & (4) \end{aligned}$$

The energy density in an electric field is

$$u=\frac{\epsilon_0}{2}E^2 \tag{5}$$

so if the plates contract by a distance ϵ then the amount of energy lost is

$$\Delta U=\frac{\epsilon_0}{2}E^2\epsilon A \tag{6}$$

so the energy from the electric field is used to move the plates closer together.

We can invert the argument by starting with 6 and arguing that because of conservation of energy, the energy lost in the field must provide the work done in moving the plates, from which we can get the force on the plates as

$$F = \frac{\Delta U}{\epsilon} = \frac{\epsilon_0}{2} E^2 A \tag{7}$$

and the pressure as

$$P = \frac{F}{A} = \frac{\epsilon_0}{2} E^2 \tag{8}$$