POTENTIAL OF TWO COPPER PIPES

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Problem 3.11.

Time to fill in a gap that I had overlooked. This problem is given in Griffiths as belonging to the method of images, but it's not really an image problem. It goes like this: We have two infinite copper pipes, each of radius R. The axis of one pipe is on the line $x=-d$, $(y=0)$ and the other is on $x=+d$. The potential of the one on the left is held at $-V_0$ and the one on the right at $+V_0$. We are to find the potential everywhere.

The solution doesn't really involve images, in that no part of space is outside the realm in which we want the answer. What it does do is exploit the uniqueness of solutions to Laplace's equation. If we look back at the problem of two charged wires, we found that if we had a linear charge density of $-\lambda$ on a wire at $x=-a$ and a density of $+\lambda$ at $x=+a$, the potential is

$$V=\frac{\lambda}{4\pi\varepsilon_0}\ln\frac{(x+a)^2+z^2}{(x-a)^2+z^2} \tag{1}$$

We also found that the equipotential surfaces (where V is a constant) are cylinders with axes at

$$x=\frac{a}{\tanh\left(2\pi\varepsilon_0 V/\lambda\right)} \tag{2}$$

and radii of

$$r=\frac{a}{\left|\sinh\left(2\pi\varepsilon_0 V/\lambda\right)\right|} \tag{3}$$

Therefore, the copper pipe problem is equivalent to a double-wire problem if we can find a pair of wires that have the equipotential surfaces specified above. That is, we must have an equipotential of $V=+V_0$ for a cylinder with axis $x=+d$ and radius $r=R$, and an equipotential of $V=-V_0$ for a cylinder with axis $x=-d$ and radius $r=R$. This means we need to find a and λ satisfying these conditions. So

$$d = \frac{a}{\tanh\left(2\pi\varepsilon_0 V_0/\lambda\right)} \tag{4}$$

$$R = \frac{a}{\sinh\left(2\pi\varepsilon_0 V_0/\lambda\right)} \tag{5}$$

From which we get

$$\frac{d}{R} = \frac{\sinh\left(2\pi\varepsilon_0 V_0/\lambda\right)}{\tanh\left(2\pi\varepsilon_0 V_0/\lambda\right)} = \cosh\left(2\pi\varepsilon_0 V_0/\lambda\right) \tag{6}$$

$$\lambda = \frac{2\pi\varepsilon_0 V_0}{\cosh^{-1}\frac{d}{R}} \tag{7}$$

$$a = R\sinh\left(2\pi\varepsilon_0 V_0/\lambda\right) \tag{8}$$

$$= R\sqrt{\cosh^2\left(2\pi\varepsilon_0 V_0/\lambda\right) - 1} \tag{9}$$

$$= \sqrt{d^2 - R^2} \tag{10}$$

The equipotential $V = -V_0$ gives us the cylinder with axis at $x = -a$. With these substitutions, the potential is given by 1.

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