POLARIZABILITY OF HYDROGEN

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Section 4.1.2; Problem 4.1.

When a neutral atom is placed in an electric field, the positive nucleus gets pulled in the direction of the field, and the negative electron cloud gets pulled in the direction opposite to the field. This results in the centres of positive and negative charge no longer being in the same place, so the atom obtains a small dipole moment. Experimentally, it is found that for small fields (that is, fields small compared to the field required to ionize the atom by removing one or more electrons), this dipole moment is approximately proportional to the applied field:

$$\mathbf{p} = \alpha \mathbf{E} \tag{1}$$

where the constant α is called the *atomic polarizability*, and has a characteristic value for each type of atom.

A crude model of polarizability assumes that the electron cloud is a uniformly charged sphere and the nucleus is a point charge. An external field is assumed to displace the cloud and nucleus so that the distance from the centre of the cloud (which is assumed still to be spherical) to the nucleus is d. We can find d by equating the force between the cloud and nucleus to the opposite of the external force from the applied field.

We've seen (Example 2 in here) that the field a distance d from the centre of a uniformly charged sphere with charge density ρ is

$$E = \frac{d\rho}{3\epsilon_0} \tag{2}$$

If the total charge in the cloud is q then $\rho = 3q/4\pi a^3$ where a is the radius of the cloud, so we can write the field as

$$E = \frac{qd}{4\pi\epsilon_0 a^3} \tag{3}$$

$$= \frac{p}{4\pi\epsilon_0 a^3} \tag{4}$$

where p is the induced dipole moment. Therefore

$$
\begin{aligned}
p &= 4\pi\epsilon_0 a^3 E \qquad &(5) \\
\alpha &= 4\pi\epsilon_0 a^3 \qquad &(6)
\end{aligned}
$$

and we have a crude formula for the atomic polarizability. Experimentally, this turns out to accurate to within a factor of 4 or so, which may not sound that great, but at least it's within an order of magnitude, and the model *is* very crude. For example, for hydrogen, if we take $a = 5.29 \times 10^{-11}$ m (the Bohr radius), we get

$$
\frac{\alpha}{4\pi\epsilon_0} = 1.48 \times 10^{-31} \ m^3 \qquad (7)
$$

Experimentally, $\alpha/4\pi\epsilon_0$ for hydrogen is $6.67 \times 10^{-31} \ m^3$.

As an example, suppose we have a hydrogen atom between a pair of metal plates 1 mm apart, with a potential difference of 500 V across the gap. In that case, the field is $E = 5 \times 10^5$ V/m. Using the experimental value for the polarizability, we get

$$
\begin{aligned}
p &= qd \qquad &(8) \\
&= \alpha E \qquad &(9) \\
&= 4\pi\epsilon_0 \times 6.67 \times 10^{-31} \times 5 \times 10^5 \qquad &(10) \\
&= 3.71 \times 10^{-35} \ C \cdot m \qquad &(11)
\end{aligned}
$$

using $4\pi\epsilon_0 = 1.11 \times 10^{-10}$ in MKS units.

With the electron charge of 1.6×10^{-19} coulombs, we get

$$
d = 2.32 \times 10^{-16} \ m \qquad (12)
$$

To estimate the voltage required to ionize the hydrogen atom, we can work out the field required to make $d = a$. That is

$$
\begin{aligned}
E_{ion} &= \frac{qa}{\alpha} \qquad &(13) \\
&= \frac{\left(1.6 \times 10^{-19}\right)\left(5.29 \times 10^{-11}\right)}{\left(6.67 \times 10^{-31}\right)\left(1.11 \times 10^{-10}\right)} \qquad &(14) \\
&= 1.14 \times 10^{11} \ V/m \qquad &(15)
\end{aligned}
$$

With plates 1 mm apart, we would therefore need a potential difference of 1.14×10^8 volts.

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