FIELD OF A POLARIZED OBJECT - EXAMPLES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problems 4.11, 4.12.

We saw before that we can write the potential of a polarized object as

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{1}{|\mathbf{r}-\mathbf{r}'|^2}\frac{(\mathbf{r}-\mathbf{r}')\cdot\mathbf{P}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3\mathbf{r}' \tag{0.1}$$

Note that this is a volume integral over the *primed* coordinates $\mathbf{r}'$, that is, over the location of the volume element containing the polarized material.

Although we can work out the field due to a uniformly polarized sphere using the techniques in the last post, it is also possible to do this using this integral directly. For a uniformly polarized sphere, $\mathbf{P}(\mathbf{r}')$ is constant over the volume of the sphere, so we can take it outside the integral.

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\mathbf{P}\cdot\int \frac{1}{|\mathbf{r}-\mathbf{r}'|^2}\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3\mathbf{r}' \tag{0.2}$$

The remaining integral depends only on the position $\mathbf{r}$ of the observation point, and not on the polarization vector. We can therefore take $\mathbf{r}$ to lie on the z axis. The angle between $\mathbf{r}$ and $\mathbf{r}'$ is therefore the polar angle θ. By symmetry, only the z component of the vector in the integral will be non-zero, so we can work out that on its own. This is actually the same problem we faced when calculating the electric field of a uniformly charged sphere, which was done here. Quoting the answer from that post, we have

$$\int \frac{1}{|\mathbf{r}-\mathbf{r}'|^2}\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3\mathbf{r}' = \begin{cases} \pm\frac{4\pi R^3}{3r^2}\hat{\mathbf{z}} & r > R \\ \pm\frac{4\pi r}{3}\hat{\mathbf{z}} & r < R \end{cases} \tag{0.3}$$

where the plus sign is taken if $\mathbf{r}$ points in the $+z$ direction and the minus sign in the other case.

We can now insert the polarization vector so that it makes an angle θ with the observation vector $\mathbf{r}$ and since $\mathbf{P}\cdot\hat{\mathbf{z}} = P\cos\theta$ we have

$$V(\mathbf{r})=\begin{cases}\frac{R^3}{3\varepsilon_0 r^2}P\cos\theta & r>R\\ \frac{r}{3\varepsilon_0}P\cos\theta & r<R\end{cases} \tag{0.4}$$

Note that the electric field inside the sphere is uniform. Since $z=r\cos\theta$:

$$\mathbf{E} = -\nabla V_{in} \tag{0.5}$$

$$= -\frac{P}{3\varepsilon_0}\hat{\mathbf{z}} \tag{0.6}$$

As a simple example of the use of the bound charges representation of a polarized object, we can look at a cylinder which contains a uniform polarization $\mathbf{P}$ parallel to its axis. In this case, $\nabla\cdot\mathbf{P}=0$ everywhere inside the cylinder and $\mathbf{P}\cdot\hat{\mathbf{n}}=0$ on the sides of the cylinder. On the ends, $\mathbf{P}\cdot\hat{\mathbf{n}}=\pm P$, so the only bound charge is on the ends. We therefore have two charged disks separated by the length L of the cylinder. We've worked out an approximation for the potential of a single charged disk before, but it's difficult to get much of a picture of what the field looks like from that solution. However, if the length is much greater than the radius, we have essentially two point charges of equal and opposite sign, which is a physical dipole. For L much less than the radius, we have a dipole layer, so we would expect field lines coming out of the side with positive charge and curling round to enter on the negative side. If the radius and length are roughly equal, we would expect field lines going directly between the disks, with lines curving round the outside as well.

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