ELECTRIC FIELD WITHIN A CAVITY INSIDE A DIELECTRIC

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References: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 4.16

We can use superposition to work out the electric field within a small cavity that is hollowed out inside a block of dielectric. We'll take the electric field within the dielectric to be $\mathbf{E}_0$ (which need not be constant), and the polarization to be $\mathbf{P}$ (again, not necessarily constant). The displacement is then $\mathbf{D}_0 = \varepsilon_0\mathbf{E}_0 + \mathbf{P}$. In the small region of interest, we'll take $\mathbf{P}$ pointing upwards, so that the field it induces points downwards.

Now if we hollow out a small sphere (assumed to be small enough that the field and polarization can be taken as constant within it), we can work out the field within the hollow sphere. The trick is that the polarization within empty space must be zero, so we can simulate the situation by superimposing a sphere with equal and opposite polarization $-\mathbf{P}$ on top of the dielectric. We've worked out the field within a uniformly polarized sphere earlier, and with polarization $-\mathbf{P}$ this field is

$$\mathbf{E}_s = \frac{1}{3\varepsilon_0}\mathbf{P} \tag{1}$$

Thus the net field within a spherical cavity is

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_s = \mathbf{E}_0 + \frac{1}{3\varepsilon_0}\mathbf{P} \tag{2}$$

Since the net polarization within the cavity is zero, the displacement is

$$\mathbf{D} = \varepsilon_0\mathbf{E}_0 + \frac{1}{3}\mathbf{P} = \mathbf{D}_0 - \frac{2}{3}\mathbf{P} \tag{3}$$

Now suppose the cavity is shaped like a long thin needle parallel to $\mathbf{P}$. Again, we superimpose a needle with the opposite polarization. We can work out the bound charges, and since the polarization within the needle is constant, $\rho_b = \nabla\cdot(-\mathbf{P}) = 0$ and since the polarization is parallel to the axis, $\sigma_b = -\mathbf{P}\cdot\hat{\mathbf{n}} = 0$ on the sides of the needle. On the ends of the needle, σ_b is non-zero, but if the needle is long enough with small end points, this will contribute a very small field so we can approximate the situation by saying that the electric field is not significantly modified within the cavity:

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_0 & (4) \\ \mathbf{D} &= \varepsilon_0\mathbf{E}_0 = \mathbf{D}_0 - \mathbf{P} & (5) \end{aligned}$$

Finally, we consider a thin, circular wafer shaped cavity perpendicular to the polarization. In this case, the bound volume charge is again zero, but the surface charge is

$$\sigma_b = -\mathbf{P}\cdot\hat{\mathbf{n}} = \pm P \tag{6}$$

with the plus sign on the bottom of the wafer and the minus sign on top. The field due to the bottom (positive) surface is (by Gauss's law) $P/2\varepsilon_0$ upwards, and the field due to the top (negative) surface is also $P/2\varepsilon_0$ upwards, so the total field within the wafer is

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_0 + \frac{1}{\varepsilon_0}\mathbf{P} & (7) \\ \mathbf{D} &= \varepsilon_0\mathbf{E} = \mathbf{D}_0 & (8) \end{aligned}$$

Thus the needle leaves $\mathbf{E}$ unchanged and the wafer leaves $\mathbf{D}$ unchanged.

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