DIELECTRIC CONSTANT

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Section 4.4.1 and Problem 4.18.

Experimentally, the dipole moment of an atom is proportional to the applied electric field (for small fields). Since the polarization of a dielectric is due to individual atoms within the dielectric being given dipole moments, it should come as no surprise that the polarization density $\mathbf{P}$ of a dielectric is also proportional to the applied field. The relation is written as

$$\mathbf{P}=\varepsilon_0\chi_e\mathbf{E} \tag{1}$$

where χ_e is the *electric susceptibility*. Due to the presence of the ε_0, χ_e is dimensionless.

Not all substances obey such a simple law, but for those that do, they are called *linear dielectrics.*

If this condition holds, we get a simple relationship between the displacement, the field and the polarization. We have

$$\begin{aligned}\mathbf{D} &= \varepsilon_0\mathbf{E}+\mathbf{P} \qquad (2)\\ &= \varepsilon_0\left(1+\chi_e\right)\mathbf{E} \qquad (3)\end{aligned}$$

The quantity $1+\chi_e$ is called the *dielectric constant* for a material. The entire proportionality constant is called the *permittivity* ε of the material:

$$\varepsilon=\varepsilon_0\left(1+\chi_e\right) \tag{4}$$

Since a vacuum cannot be polarized at all, $\chi_e=0$ for a vacuum, so that $\varepsilon=\varepsilon_0$ in that case. This is why ε_0 is called the *permittivity of free space*.

As a simple example of how these relations can be used, suppose we have a parallel plate capacitor whose plates are separated by a distance $2a$. On one plate there is a surface charge density of σ and on the other there is $-\sigma$. Between the plates are two slabs of dielectric, each of thickness a. Slab 1 (next to the positive plate) has a dielectric constant of 2 and slab 2 has a dielectric constant of 1.5.

We can begin by finding the displacement $\mathbf{D}$. Using Gauss's law for displacement, we can build a little cylindrical Gaussian pillbox of radius r with one end in the positive plate and the other in slab 1. By symmetry $\mathbf{D}$ is parallel to the axis of the cylinder so there are no contributions to $\mathbf{D}\cdot d\mathbf{a}$ from the sides of the cylinder. The end of the cylinder inside the plate will have $\mathbf{D}=0$ (since we're inside a conductor), while the other end will contribute $\pi r^2 D$. The charge enclosed by the cylinder is $\pi r^2\sigma$ so we get

$$\int \mathbf{D}\cdot d\mathbf{a} = Q \tag{5}$$

$$\pi r^2 D = \pi r^2 \sigma \tag{6}$$

$$D = \sigma \tag{7}$$

This result is independent of the dielectric, so it holds everywhere between the plates. If we assume the positive plate lies above the negative one, we have $\mathbf{D}=-\sigma\hat{\mathbf{z}}$ since the displacement vector points from positive to negative.

From the relation above, we can find $\mathbf{E}$. For the two slabs, we have

$$\mathbf{E}_1 = \frac{1}{\varepsilon_0\left(1+\chi_e\right)_1}\mathbf{D} \tag{8}$$

$$= -\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{z}} \tag{9}$$

$$\mathbf{E}_2 = -\frac{\sigma}{1.5\varepsilon_0}\hat{\mathbf{z}} \tag{10}$$

The polarization within the two slabs can now be found from 1.

$$\mathbf{P}_1 = \varepsilon_0(2-1)\mathbf{E}_1 \tag{11}$$

$$= -\frac{\sigma}{2}\hat{\mathbf{z}} \tag{12}$$

$$\mathbf{P}_2 = -\frac{\sigma}{3}\hat{\mathbf{z}} \tag{13}$$

The potential difference between the plates is

$$V = E_1 a + E_2 a \tag{14}$$

$$= \frac{7}{6}\frac{a\sigma}{\varepsilon_0} \tag{15}$$

The bound charges resulting from the polarization can be found. Since the polarization is uniform within each slab, there is no volume bound charge since $\nabla\cdot\mathbf{P}=0$ everywhere. The surface bound charge is found from

$$\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}} \tag{16}$$

Applying this at the boundary between the positive plate and slab 1, we get $\sigma_b = -\sigma/2$. At the boundary between slabs 1 and 2, there is a bound charge of $+\sigma/2$ on slab 1 and $-\sigma/3$ on slab 2 (for a net bound charge of $+\sigma/6$ at the boundary). At the boundary between slab 2 and the negative plate, we have $+\sigma/3$.

We can use these bound charges to check the values for the field found above. Choosing a Gaussian cylinder with one end in the positive plate and the other in slab 1, the enclosed charge is $\pi r^2(\sigma-\sigma/2)$ which is equal to ϵ_0 times the surface integral of the field over the cylinder's end cap which is $\pi r^2 E_1$. Thus $E_1 = \sigma/2\epsilon_0$ (pointing downwards) as before. Doing the same calculation for slab 2 we get $E_2 = 2\sigma/3\epsilon_0$ as before.

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