

DIELECTRIC SPHERE WITH FREE CHARGE

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the auxiliary blog.

Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Section 4.4.1 and Problem 4.20.

Suppose we have a sphere of linear dielectric (radius R) in which is embedded a uniform free charge of density ρ .

From the definition of displacement, we have that

$$(1) \quad \nabla \cdot \mathbf{D} = \rho$$

The integral form of this is, using Gauss's law:

$$(2) \quad \int \mathbf{D} \cdot d\mathbf{a} = Q$$

where Q is the charge enclosed by the surface over which the integral is done. In this case, from symmetry, we get for points $r < R$:

$$(3) \quad 4\pi r^2 D = \frac{4}{3}\pi r^3 \rho$$

$$(4) \quad D = \frac{r}{3}\rho$$

Outside the sphere, we have

$$(5) \quad 4\pi r^2 D = \frac{4}{3}\pi R^3 \rho$$

$$(6) \quad D = \frac{R^3}{3r^2}\rho$$

In both regions, the relation between D and E is $D = \epsilon_0 \epsilon_r E$ where ϵ_r is the dielectric constant, so we have

$$(7) \quad E = \begin{cases} \frac{r\rho}{3\epsilon_0 \epsilon_r} & 0 < r < R \\ \frac{R^3 \rho}{3\epsilon_0 r^2} & r > R \end{cases}$$

Outside the sphere the dielectric constant is 1, since we are in a vacuum there.

From this we can find the potential at the centre of the sphere, relative to infinity.

$$(8) \quad V = - \int_{\infty}^0 E dr$$

$$(9) \quad = - \frac{R^3 \rho}{3\epsilon_0} \int_{\infty}^R \frac{dr}{r^2} - \frac{\rho}{3\epsilon_0 \epsilon_r} \int_R^0 r dr$$

$$(10) \quad = \frac{R^2 \rho}{3\epsilon_0} + \frac{R^2 \rho}{6\epsilon_0 \epsilon_r}$$

$$(11) \quad = \frac{R^2 \rho}{3\epsilon_0} \left[1 + \frac{1}{2\epsilon_r} \right]$$