COAXIAL CABLE WITH DIELECTRIC

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 4.21.

Suppose we have a coaxial cable with an inner conducting core of radius a and an outer conducting cylinder of radius c. Part of the space between the conductors is filled with a linear dielectric with dielectric constant ϵ_r which extends from radius b to c.

We can use an analysis similar to that which we used in working out the capacitance of a coaxial cable earlier. If we place a surface charge density of σ on the inner conductor, then using the analysis from before, the potential in the region $a<r<b$ is

$$V=\frac{a\sigma}{\epsilon_0}\ln\frac{r}{a} \tag{1}$$

For the region $b<r<c$, the electric field is reduced by a factor of ϵ_r so in this region we have for the potential relative to radius b:

$$V=\frac{a\sigma}{\epsilon_0\epsilon_r}\ln\frac{r}{b} \tag{2}$$

The total potential difference between the two conductors is then

$$\begin{aligned}
V_{tot} &= \frac{a\sigma}{\epsilon_0}\ln\frac{b}{a}+\frac{a\sigma}{\epsilon_0\epsilon_r}\ln\frac{c}{b} && (3)\\
&= \frac{a\sigma}{\epsilon_0}\left[\ln\frac{b}{a}+\frac{1}{\epsilon_r}\ln\frac{c}{b}\right] && (4)
\end{aligned}$$

The capacitance per unit length of the cable is found from $C=Q/V$. The charge per unit length is $Q=2\pi a\sigma$, so we get

$$\begin{aligned}
C &= \frac{2\pi a\sigma}{\frac{a\sigma}{\epsilon_0}\left[\ln\frac{b}{a}+\frac{1}{\epsilon_r}\ln\frac{c}{b}\right]} && (5)\\
&= \frac{2\pi\epsilon_0}{\ln\frac{b}{a}+\frac{1}{\epsilon_r}\ln\frac{c}{b}} && (6)
\end{aligned}$$