

COAXIAL CABLE WITH DIELECTRIC

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 4.21.

Suppose we have a coaxial cable with an inner conducting core of radius a and an outer conducting cylinder of radius c . Part of the space between the conductors is filled with a linear dielectric with dielectric constant ϵ_r , which extends from radius b to c .

We can use an analysis similar to that which we used in working out the capacitance of a coaxial cable earlier. If we place a surface charge density of σ on the inner conductor, then using the analysis from before, the potential in the region $a < r < b$ is

$$(1) \quad V = \frac{a\sigma}{\epsilon_0} \ln \frac{r}{a}$$

For the region $b < r < c$, the electric field is reduced by a factor of ϵ_r , so in this region we have for the potential relative to radius b :

$$(2) \quad V = \frac{a\sigma}{\epsilon_0 \epsilon_r} \ln \frac{r}{b}$$

The total potential difference between the two conductors is then

$$(3) \quad V_{tot} = \frac{a\sigma}{\epsilon_0} \ln \frac{b}{a} + \frac{a\sigma}{\epsilon_0 \epsilon_r} \ln \frac{c}{b}$$

$$(4) \quad = \frac{a\sigma}{\epsilon_0} \left[\ln \frac{b}{a} + \frac{1}{\epsilon_r} \ln \frac{c}{b} \right]$$

The capacitance per unit length of the cable is found from $C = Q/V$. The charge per unit length is $Q = 2\pi a\sigma$, so we get

$$(5) \quad C = \frac{2\pi a\sigma}{\frac{a\sigma}{\epsilon_0} \left[\ln \frac{b}{a} + \frac{1}{\epsilon_r} \ln \frac{c}{b} \right]}$$

$$(6) \quad = \frac{2\pi\epsilon_0}{\ln \frac{b}{a} + \frac{1}{\epsilon_r} \ln \frac{c}{b}}$$