DIELECTRIC SPHERE IN UNIFORM ELECTRIC FIELD

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 4.23.

We can work out the field inside a sphere of dielectric placed in a uniform electric field in the same way we tackled the cylinder in the last post (for those interested, this is done in Griffiths's book). However, we can also use a method of successive approximations to get the answer.

The starting point is to assume that the uniform exterior field $\mathbf{E}_0$ covers all space, including the interior of the sphere. We then know that this uniform field will cause the sphere to become polarized. Since we're dealing with a linear dielectric, this polarization is

$$\mathbf{P}_0 = \varepsilon_0 \chi_e \mathbf{E}_0 \tag{1}$$

However, a uniformly polarized sphere produces its own electric field, which we need to add onto the original uniform field. We worked out the field inside a uniformly polarized sphere earlier, so we can quote that result. We get

$$\begin{aligned} \mathbf{E}_1 &= -\frac{1}{3\varepsilon_0}\mathbf{P}_0 & (2)\\ &= -\frac{\chi_e}{3}\mathbf{E}_0 & (3) \end{aligned}$$

This new field produces more polarization, so we get

$$\begin{aligned} \mathbf{P}_1 &= \varepsilon_0 \chi_e \mathbf{E}_1 & (4)\\ &= -\varepsilon_0 \frac{\chi_e^2}{3}\mathbf{E}_0 & (5) \end{aligned}$$

Another round gives the next correction to the field:

$$\mathbf{E}_2 = \frac{\chi_e^2}{3^2}\mathbf{E}_0 \tag{6}$$

It's fairly obvious that the general pattern is

$$\mathbf{E}_n = (-1)^n \left(\frac{\chi_e}{3}\right)^n \mathbf{E}_0 \tag{7}$$

so the total field is the sum of all these increments:

$$\mathbf{E}=\sum_{n=0}^{\infty}(-1)^{n}\left(\frac{\chi_{e}}{3}\right)^{n}\mathbf{E}_{0} \tag{8}$$

The sum is a geometric series, so we can sum it using the standard formula and we get

$$\begin{aligned}\mathbf{E} &= \frac{1}{1+\chi_{e}/3}\mathbf{E}_{0} && (9)\\ &= \frac{3}{2+\varepsilon_{r}}\mathbf{E}_{0} && (10)\end{aligned}$$

where $\varepsilon_{r}=1+\chi_{e}$ is the dielectric constant. This agrees with the result obtained using Legendre polynomials, as given in Griffiths.

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