

DIELECTRIC SPHERE IN UNIFORM ELECTRIC FIELD

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 4.23.

We can work out the field inside a sphere of dielectric placed in a uniform electric field in the same way we tackled the cylinder in the last post (for those interested, this is done in Griffiths's book). However, we can also use a method of successive approximations to get the answer.

The starting point is to assume that the uniform exterior field \mathbf{E}_0 covers all space, including the interior of the sphere. We then know that this uniform field will cause the sphere to become polarized. Since we're dealing with a linear dielectric, this polarization is

$$(0.1) \quad \mathbf{P}_0 = \epsilon_0 \chi_e \mathbf{E}_0$$

However, a uniformly polarized sphere produces its own electric field, which we need to add onto the original uniform field. We worked out the field inside a uniformly polarized sphere earlier, so we can quote that result. We get

$$(0.2) \quad \mathbf{E}_1 = -\frac{1}{3\epsilon_0} \mathbf{P}_0$$

$$(0.3) \quad = -\frac{\chi_e}{3} \mathbf{E}_0$$

This new field produces more polarization, so we get

$$(0.4) \quad \mathbf{P}_1 = \epsilon_0 \chi_e \mathbf{E}_1$$

$$(0.5) \quad = -\epsilon_0 \frac{\chi_e^2}{3} \mathbf{E}_0$$

Another round gives the next correction to the field:

$$(0.6) \quad \mathbf{E}_2 = \frac{\chi_e^2}{3^2} \mathbf{E}_0$$

It's fairly obvious that the general pattern is

$$(0.7) \quad \mathbf{E}_n = (-1)^n \left(\frac{\chi_e}{3}\right)^n \mathbf{E}_0$$

so the total field is the sum of all these increments:

$$(0.8) \quad \mathbf{E} = \sum_{n=0}^{\infty} (-1)^n \left(\frac{\chi_e}{3}\right)^n \mathbf{E}_0$$

The sum is a geometric series, so we can sum it using the standard formula and we get

$$(0.9) \quad \mathbf{E} = \frac{1}{1 + \chi_e/3} \mathbf{E}_0$$

$$(0.10) \quad = \frac{3}{2 + \epsilon_r} \mathbf{E}_0$$

where $\epsilon_r = 1 + \chi_e$ is the dielectric constant. This agrees with the result obtained using Legendre polynomials, as given in Griffiths.

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