DIELECTRIC SPHERE IN UNIFORM ELECTRIC FIELD

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We can work out the field inside a sphere of dielectric placed in a uniform electric field in the same way we tackled the cylinder in the last post (for those interested, this is done in Griffiths’s book). However, we can also use a method of successive approximations to get the answer.

The starting point is to assume that the uniform exterior field $E_0$ covers all space, including the interior of the sphere. We then know that this uniform field will cause the sphere to become polarized. Since we’re dealing with a linear dielectric, this polarization is

$$P_0 = \epsilon_0 \chi E_0$$  \hspace{1cm} (1)

However, a uniformly polarized sphere produces its own electric field, which we need to add onto the original uniform field. We worked out the field inside a uniformly polarized sphere earlier, so we can quote that result. We get

$$E_1 = -\frac{1}{3\epsilon_0} P_0 \hspace{1cm} (2)$$

$$= -\frac{\chi e}{3} E_0 \hspace{1cm} (3)$$

This new field produces more polarization, so we get

$$P_1 = \epsilon_0 \chi_e E_1 \hspace{1cm} (4)$$

$$= -\epsilon_0 \frac{\chi e}{3} E_0 \hspace{1cm} (5)$$

Another round gives the next correction to the field:

$$E_2 = \frac{\chi e}{3^2} E_0$$  \hspace{1cm} (6)

It’s fairly obvious that the general pattern is
\[ E_n = (-1)^n \left( \frac{\chi_e}{3} \right)^n E_0 \]  

so the total field is the sum of all these increments:

\[ E = \sum_{n=0}^{\infty} (-1)^n \left( \frac{\chi_e}{3} \right)^n E_0 \]  

The sum is a geometric series, so we can sum it using the standard formula and we get

\[ E = \frac{1}{1 + \chi_e/3} E_0 \]  

\[ = \frac{3}{2 + \epsilon_r} E_0 \]

where \( \epsilon_r = 1 + \chi_e \) is the dielectric constant. This agrees with the result obtained using Legendre polynomials, as given in Griffiths.

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