FORCE ON A DIELECTRIC

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Section 4.4.4 & Problem 4.28.

We've seen that a conductor carrying some free charge experiences electrostatic pressure due to the interaction of the free charges. A dielectric in an external electric field also experiences a force from that field.

In the case of a parallel plate capacitor filled with dielectric, the field over most of the plates' area is perpendicular to the plates and thus there is no net force on the dielectric since it is held in place between the plates and the electric force is balanced by the mechanical force exerted by the plates. However, at the edges of the plates, there is a fringing field which curves outwards from the plates themselves, and this tends to pull the dielectric further into the capacitor. If the dielectric fills the capacitor, then the forces at all the edges balance each other and the dielectric experiences no net force. However, if the dielectric only partially fills the space between the plates, there is a net force tending to pull the dielectric further in.

A nice example of this is the following problem. We have two concentric cylindrical tubes. The inner tube has radius a and the outer tube has radius b. The tubes are placed vertically into a dish of dielectric oil with susceptibility χ_e and mass density ρ, and a constant voltage of V is applied between the two tubes. The oil will be attracted into the space between the tubes, so it will tend to rise to a height h.

We've already seen that the capacitance per unit length of a coaxial pair of cylinders in a vacuum is

$$C=\frac{2\pi\epsilon_0}{\ln(b/a)} \tag{1}$$

If the total length of the cylinder is L, and the height of the oil is h, then the portions of the cylinder will have capacitances of

$$C_{oil} = \frac{2\pi\epsilon_0\epsilon_r}{\ln(b/a)}h \tag{2}$$

$$C_{vac} = \frac{2\pi\epsilon_0}{\ln(b/a)}(L-h) \tag{3}$$

(Note that the capacitance of the region with the oil is increased by a factor of the dielectric constant $\varepsilon_r = 1 + \chi_e$.) The total capacitance is thus

$$C = \frac{2\pi\varepsilon_0}{\ln(b/a)}\left[\varepsilon_r h + L - h\right] \tag{4}$$

$$= \frac{2\pi\varepsilon_0}{\ln(b/a)}\left(\chi_e h + L\right) \tag{5}$$

Since we're holding the voltage constant and the capacitance increases as the oil rises, the amount of charge on the plates must also increase. That is, we must transfer charge from the inner to the outer tube in order to maintain a constant V. The amount of work required to transfer an amount of charge dq through a potential difference V is Vdq. There are thus two sources of work done here: the energy required to make the oil rise between the plates, and the energy required to transfer charge from one tube to the other.

If we choose instead to assume that the charge on the plates rather than the voltage remains constant then no charge is transferred so no work is done by the Vdq term, which is now zero. In this case, the voltage would change as the oil rises. However, since the oil would still rise, work is still being done.

Since the energy stored in a capacitor is $W = \frac{1}{2}CV^2$, if V is constant, the change in energy if we move the oil a distance dh is

$$dW = \frac{1}{2}V^2\frac{dC}{dh}dh \tag{6}$$

Since the force we apply to move the oil is in opposition to the electrical force F we get

$$F = -\frac{dW}{dh} \tag{7}$$

$$= -\frac{1}{2}V^2\frac{dC}{dh} \tag{8}$$

However, in the constant voltage problem, we also have to move a charge dq from one plate to the other, requiring work Vdq. This adds another term onto the force, which is, since $q = CV$:

$$F_q = V\frac{dq}{dh} \tag{9}$$

$$= V^2\frac{dC}{dh} \tag{10}$$

This results in a total force of

$$F_t = F + F_q \tag{11}$$

$$= \frac{1}{2}V^2\frac{dC}{dh} \tag{12}$$

It's worth looking at this in a bit more detail. The capacitance C increases as the dielectric constant ε_r increases, so if we keep the charge Q on the capacitor constant as the oil rises (and thus increases the dielectric constant over a greater range, increasing C) then since $Q = CV$, V must decrease. The energy stored in the capacitor is $W = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$ therefore also decreases. This energy goes into the work done to raise the oil.

However, if we keep V constant as the oil rises, then from $Q = CV$, Q must increase as C increases. The energy stored in the capacitor is $W = \frac{1}{2}CV^2$ and thus it *increases*. The extra energy is provided by an external battery which adds charge to the capacitor in order to maintain a constant voltage, as well as provide the energy to raise the oil. This is the extra $V\,dq$ term.

Returning to the problem of determining the height of the oil, the electrical force on the oil is balanced by the gravitational force, so we get

$$\frac{1}{2}V^2\frac{dC}{dh} = mg \tag{13}$$

$$\frac{1}{2}V^2\frac{2\pi\varepsilon_0\chi_e}{\ln(b/a)} = \pi\left(b^2 - a^2\right)\rho g h \tag{14}$$

$$h = \frac{\varepsilon_0\chi_e V^2}{\rho g\left(b^2 - a^2\right)\ln(b/a)} \tag{15}$$

PINGBACKS

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