DIPOLE BETWEEN TWO ANGLED CONDUCTING PLANES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 4.30.

Suppose we have two conducting planes that make angles of $+\theta$ and $-\theta$ respectively with the xy plane, and meet along the y axis (although they are insulated from each other). The top plane is held at a potential of $+V$ and the bottom plane at $-V$.

Since the electric field is always normal to the surface of a conductor, the field lines will bulge outwards in the $+x$ direction and have a vertical tangent (that is, parallel to the z axis) when they pass through the xy plane. Since the field lines travel from positive to negative, the field lines start on the top plane and arc towards the bottom plane.

Now suppose we place a dipole between the planes. The dipole is centred on the xy plane and the dipole moment $\mathbf{p}$ points in the $+z$ direction. What force does this dipole feel?

This depends on whether we are talking about a physical dipole (that is, two charges separated by a finite distance) or an ideal dipole, in which the separation is zero. We've seen that the force felt by an ideal dipole in an electric field is

$$\mathbf{F}=(\mathbf{p}\cdot\nabla)\mathbf{E} \tag{0.1}$$

Since $\mathbf{p}$ is parallel to $\hat{\mathbf{z}}$, $\mathbf{F}=p\frac{\partial}{\partial z}\mathbf{E}$ and since $\mathbf{E}$ is tangent to the vertical at the location of the dipole, $\partial\mathbf{E}/\partial z=0$ and the force is zero.

If we're talking about a physical dipole, then the positive charge is slightly above the xy plane where the field points slightly to the right of $-\hat{\mathbf{z}}$, so the force on the positive charge will be in that direction. Similarly, the negative charge is slightly below the xy plane, where the field points slightly to the left of $-\hat{\mathbf{z}}$. The force on the negative charge will thus be in the opposite direction, or slightly to the right of $+\hat{\mathbf{z}}$. From the symmetry of the problem, the vertical components of force cancel and the dipole feels a net force to the right.