DIELECTRIC CUBE: BOUND CHARGES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 4.31.

Another example of the bound charge due to polarization in a dielectric. Suppose we have a dielectric cube of side length a centred at the origin, with a frozen-in polarization of $\mathbf{P}=k\mathbf{r}$, for some constant k. The bound charges induced by this polarization are

$$\sigma_b \equiv \mathbf{P}\cdot\hat{\mathbf{n}} \tag{1}$$
$$\rho_b \equiv -\nabla'\cdot\mathbf{P} \tag{2}$$

The volume bound charge density is

$$\rho_b = -k\nabla\cdot\mathbf{r} \tag{3}$$
$$= -3k \tag{4}$$

which is found by expressing $\mathbf{r}$ in rectangular coordinates.

Since the charge density is constant, the total volume bound charge is

$$Q_v = -3ka^3 \tag{5}$$

For the surface charge density, by symmetry all 6 faces will be the same, so we can look at the upward facing side of the cube. Here

$$\sigma_b = k\mathbf{r}\cdot\hat{\mathbf{n}} \tag{6}$$
$$= k\frac{a}{2} \tag{7}$$

since the dot product is just the projection of $\mathbf{r}$ onto the z axis which for the upper face, is always $a/2$. Again, the surface charge density is a constant, so the charge on one face is $k\frac{a}{2}a^2$ and the total charge on the surface of the cube is

$$Q_s = 6k\frac{a}{2}a^2 \tag{8}$$

$$= 3ka^3 \tag{9}$$

The total bound charge is zero, as required.