

DIELECTRIC CUBE: BOUND CHARGES

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 4.31.

Another example of the bound charge due to polarization in a dielectric. Suppose we have a dielectric cube of side length a centred at the origin, with a frozen-in polarization of $\mathbf{P} = k\mathbf{r}$, for some constant k . The bound charges induced by this polarization are

$$\sigma_b \equiv \mathbf{P} \cdot \hat{\mathbf{n}} \quad (1)$$

$$\rho_b \equiv -\nabla' \cdot \mathbf{P} \quad (2)$$

The volume bound charge density is

$$\rho_b = -k\nabla \cdot \mathbf{r} \quad (3)$$

$$= -3k \quad (4)$$

which is found by expressing \mathbf{r} in rectangular coordinates.

Since the charge density is constant, the total volume bound charge is

$$Q_v = -3ka^3 \quad (5)$$

For the surface charge density, by symmetry all 6 faces will be the same, so we can look at the upward facing side of the cube. Here

$$\sigma_b = k\mathbf{r} \cdot \hat{\mathbf{n}} \quad (6)$$

$$= k\frac{a}{2} \quad (7)$$

since the dot product is just the projection of \mathbf{r} onto the z axis which for the upper face, is always $a/2$. Again, the surface charge density is a constant, so the charge on one face is $k\frac{a}{2}a^2$ and the total charge on the surface of the cube is

$$Q_s = 6k\frac{a}{2}a^2 \quad (8)$$

$$= 3ka^3 \quad (9)$$

The total bound charge is zero, as required.