

POINT CHARGE IN DIELECTRIC SPHERE

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 4.32.

Yet another example of the bound charge due to polarization in a dielectric. Suppose we have a point charge at the centre of a sphere of linear dielectric (susceptibility χ_e) of radius R . We can begin the solution by calculating the displacement, since the system has spherical symmetry. We know that

$$\begin{aligned} (1) \quad \int_V \nabla \cdot \mathbf{D} d^3\mathbf{r} &= Q_f \\ (2) \quad &= \int_A \mathbf{D} \cdot d\mathbf{a} \end{aligned}$$

where Q_f is the free charge enclosed by the surface of integration. Here we can take the surface to be a sphere of radius r centred at the point charge, which is the only free charge in the problem. Therefore

$$\begin{aligned} (3) \quad \int_A \mathbf{D} \cdot d\mathbf{a} &= 4\pi r^2 D(r) \\ (4) \quad &= q \\ (5) \quad \mathbf{D}(\mathbf{r}) &= \frac{q}{4\pi r^2} \hat{\mathbf{r}} \end{aligned}$$

For a linear dielectric we have

$$(6) \quad \mathbf{D} = \epsilon_0 (1 + \chi_e) \mathbf{E}$$

so we can get the field

$$(7) \quad \mathbf{E} = \begin{cases} \frac{q}{4\pi\epsilon_0(1+\chi_e)r^2} \hat{\mathbf{r}} & 0 < r < R \\ \frac{q}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}} & r > R \end{cases}$$

The polarization, again because we're dealing with a linear dielectric, is

$$(8) \quad \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$$

$$(9) \quad = \frac{\chi_e q}{4\pi(1+\chi_e)} \frac{\hat{\mathbf{r}}}{r^2}$$

The bound charges induced by this polarization are

$$(10) \quad \sigma_b \equiv \mathbf{P} \cdot \hat{\mathbf{n}}$$

$$(11) \quad \rho_b \equiv -\nabla \cdot \mathbf{P}$$

At the surface of the sphere, $\hat{\mathbf{n}} = \hat{\mathbf{r}}$ and we get

$$(12) \quad \sigma_b = \frac{\chi_e q}{4\pi(1+\chi_e)R^2}$$

$$(13) \quad Q_s = 4\pi R^2 \sigma_b$$

$$(14) \quad = \frac{\chi_e q}{(1+\chi_e)}$$

For the bound charge, we make use of the formula for the three-dimensional delta function:

$$(15) \quad \nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2} \right) = 4\pi \delta_3(\mathbf{r})$$

The bound charge is therefore

$$(16) \quad \rho_b = -\nabla \cdot \mathbf{P}$$

$$(17) \quad = -\frac{\chi_e q}{(1+\chi_e)} \delta_3(\mathbf{r})$$

The total volume bound charge is the integral of this over the sphere, which is just

$$(18) \quad Q_v = -\int_V \frac{\chi_e q}{(1+\chi_e)} \delta_3(\mathbf{r}) d^3 \mathbf{r}$$

$$(19) \quad = -\frac{\chi_e q}{(1+\chi_e)}$$

All the volume bound charge is concentrated at the centre, and the total bound charge is $Q_v + Q_s = 0$ as required.