

POINT CHARGE IN DIELECTRIC SPHERE

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 4.32.

Yet another example of the bound charge due to polarization in a dielectric. Suppose we have a point charge at the centre of a sphere of linear dielectric (susceptibility χ_e) of radius R . We can begin the solution by calculating the displacement, since the system has spherical symmetry. We know that

$$\int_V \nabla \cdot \mathbf{D} d^3\mathbf{r} = Q_f \quad (1)$$

$$= \int_A \mathbf{D} \cdot d\mathbf{a} \quad (2)$$

where Q_f is the free charge enclosed by the surface of integration. Here we can take the surface to be a sphere of radius r centred at the point charge, which is the only free charge in the problem. Therefore

$$\int_A \mathbf{D} \cdot d\mathbf{a} = 4\pi r^2 D(r) \quad (3)$$

$$= q \quad (4)$$

$$\mathbf{D}(\mathbf{r}) = \frac{q}{4\pi r^2} \hat{\mathbf{r}} \quad (5)$$

For a linear dielectric we have

$$\mathbf{D} = \epsilon_0 (1 + \chi_e) \mathbf{E} \quad (6)$$

so we can get the field

$$\mathbf{E} = \begin{cases} \frac{q}{4\pi\epsilon_0(1+\chi_e)r^2} \hat{\mathbf{r}} & 0 < r < R \\ \frac{q}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}} & r > R \end{cases} \quad (7)$$

The polarization, again because we're dealing with a linear dielectric, is

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E} \quad (8)$$

$$= \frac{\chi_e q}{4\pi(1+\chi_e)r^2} \hat{\mathbf{r}} \quad (9)$$

The bound charges induced by this polarization are

$$\sigma_b \equiv \mathbf{P} \cdot \hat{\mathbf{n}} \quad (10)$$

$$\rho_b \equiv -\nabla \cdot \mathbf{P} \quad (11)$$

At the surface of the sphere, $\hat{\mathbf{n}} = \hat{\mathbf{r}}$ and we get

$$\sigma_b = \frac{\chi_e q}{4\pi(1+\chi_e)R^2} \quad (12)$$

$$Q_s = 4\pi R^2 \sigma_b \quad (13)$$

$$= \frac{\chi_e q}{(1+\chi_e)} \quad (14)$$

For the bound charge, we make use of the formula for the three-dimensional delta function:

$$\nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2} \right) = 4\pi \delta_3(\mathbf{r}) \quad (15)$$

The bound charge is therefore

$$\rho_b = -\nabla \cdot \mathbf{P} \quad (16)$$

$$= -\frac{\chi_e q}{(1+\chi_e)} \delta_3(\mathbf{r}) \quad (17)$$

The total volume bound charge is the integral of this over the sphere, which is just

$$Q_v = -\int_V \frac{\chi_e q}{(1+\chi_e)} \delta_3(\mathbf{r}) d^3\mathbf{r} \quad (18)$$

$$= -\frac{\chi_e q}{(1+\chi_e)} \quad (19)$$

All the volume bound charge is concentrated at the centre, and the total bound charge is $Q_v + Q_s = 0$ as required.