FORCE ON A DIELECTRIC SPHERE DUE TO A CHARGED WIRE

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 4.37.

The force on an ideal dipole in an electric field is

$$\mathbf{F}=(\mathbf{p}\cdot\nabla)\,\mathbf{E} \tag{0.1}$$

In a dielectric with polarization density $\mathbf{P}(\mathbf{r})$, we can integrate this to get the total force on the dielectric due to an external field:

$$\mathbf{F}=\int_{\mathscr{V}}(\mathbf{P}\cdot\nabla)\,\mathbf{E}d^3\mathbf{r} \tag{0.2}$$

where $\mathscr{V}$ is the volume occupied by the dielectric.

In the case of a sphere of linear dielectric of radius R situated a distance z from an infinite wire carrying charge density λ, we can use this formula to work out the force on the sphere. We'll assume that the sphere is very small, so that $R\ll z$.

The field due to the wire along a line perpendicular to the wire and passing through the centre of the sphere (which we're taking as the z axis) is

$$\mathbf{E}=\frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}\hat{\mathbf{z}} \tag{0.3}$$

Assuming that the sphere is small enough, we can approximate this situation by assuming that the field is constant over the sphere. In that case, we can use the formula for the field inside a dielectric sphere in a uniform field:

$$\begin{aligned}\mathbf{E}_{in} &= \frac{3}{2+\varepsilon_r}\mathbf{E} &(0.4)\\ &= \frac{3}{2+\varepsilon_r}\frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}\hat{\mathbf{z}} &(0.5)\end{aligned}$$

Since the dielectric is linear

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}_{in} \tag{0.6}$$

$$= \frac{6\lambda\chi_e}{4\pi(2+\varepsilon_r)z}\hat{\mathbf{z}} \tag{0.7}$$

The integrand above is then

$$(\mathbf{P}\cdot\nabla)\mathbf{E} = -\frac{3\lambda^2\chi_e}{4\pi^2\varepsilon_0(2+\varepsilon_r)z^3}\hat{\mathbf{z}} \tag{0.8}$$

By assuming this is constant over the sphere, we get the force as

$$\mathbf{F} \approx -\frac{4\pi R^3}{3}\frac{3\lambda^2\chi_e}{4\pi^2\varepsilon_0(2+\varepsilon_r)z^3}\hat{\mathbf{z}} \tag{0.9}$$

$$= -\frac{\chi_e\lambda^2 R^3}{\pi\varepsilon_0(3+\chi_e)z^3}\hat{\mathbf{z}} \tag{0.10}$$

The force is attractive as you'd expect since the positive charge on the wire would induce a negative surface charge on the side of the sphere facing the wire.