RELATION BETWEEN POLARIZABILITY AND SUSCEPTIBILITY

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 4.38.

The polarization density of a linear dielectric can be written as

$$\mathbf{P}=\varepsilon_0\chi_e\mathbf{E} \tag{1}$$

where $\mathbf{E}$ is the macroscopic field applied to the dielectric. From a microscopic point of view, the dipole moment of an atom is

$$\mathbf{p}=\alpha\mathbf{E}_{local} \tag{2}$$

where $\mathbf{E}_{local}$ is the external field at the location of the atom. The difference between $\mathbf{E}$ and $\mathbf{E}_{local}$ is that $\mathbf{E}_{local}$ excludes the field produced by the atomic dipole. We can use this distinction to work out the relation between α, the polarizability (a microscopic quantity), and χ_e, the susceptibility (a macroscopic quantity).

The average field for a charge distribution within a sphere of radius R can be written in terms of its dipole moment, so if we assume that an atom occupies a sphere with this radius, we have for the local field due to the atom:

$$\mathbf{E}_{atom}=-\frac{1}{4\pi\varepsilon_0R^3}\mathbf{p} \tag{3}$$

The total field at the location of the atom is then

$$\begin{aligned}
\mathbf{E} &= \mathbf{E}_{local}+\mathbf{E}_{atom} && (4)\\
&= \mathbf{E}_{local}-\frac{1}{4\pi\varepsilon_0R^3}\mathbf{p} && (5)\\
&= \left(1-\frac{\alpha}{4\pi\varepsilon_0R^3}\right)\mathbf{E}_{local} && (6)
\end{aligned}$$

We can write this in terms of the number density of atoms. If one atom occupies a sphere of radius R then the number of atoms per unit volume is

$$N=\frac{3}{4\pi R^{3}} \tag{7}$$

so

$$\mathbf{E}=\left(1-\frac{\alpha N}{3\varepsilon_{0}}\right)\mathbf{E}_{local} \tag{8}$$

From this we can get the relation between α and χ_e, since the polarization density is

$$\mathbf{P}=N\mathbf{p} \tag{9}$$
$$=N\alpha\mathbf{E}_{local} \tag{10}$$
$$\varepsilon_{0}\chi_{e}\mathbf{E}=N\alpha\mathbf{E}_{local} \tag{11}$$
$$\varepsilon_{0}\chi_{e}\left(1-\frac{\alpha N}{3\varepsilon_{0}}\right)\mathbf{E}_{local}=N\alpha\mathbf{E}_{local} \tag{12}$$
$$\chi_{e}=\frac{N\alpha/\varepsilon_{0}}{1-\alpha N/3\varepsilon_{0}} \tag{13}$$

In terms of the dielectric constant $\varepsilon_r=1+\chi_e$, so

$$\varepsilon_{r}-1 = \frac{N\alpha/\varepsilon_{0}}{1-\alpha N/3\varepsilon_{0}} \tag{14}$$
$$\varepsilon_{r}-1-\left(\varepsilon_{r}-1\right)\frac{\alpha N}{3\varepsilon_{0}} = \frac{N\alpha}{\varepsilon_{0}} \tag{15}$$
$$\alpha = \frac{3\varepsilon_{0}}{N}\frac{\varepsilon_{r}-1}{\varepsilon_{r}+2} \tag{16}$$

This last equation is known as the Clausius-Mossotti formula, and can be used to work out the polarizability if the dielectric constant and number density are known. The dielectric constant can usually be measured fairly easily, and at least for gases, the number density can be found from the ideal gas law.

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