CURRENTS IN MAGNETIC FIELDS

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Reference: Griffiths, David J. (2007) Introduction to Electrodynamics, 3rd Edition; Prentice Hall - Problem 5.4.

Because an electric current involves moving charge, it feels a force when a magnetic field is applied to it. Currents also *produce* magnetic fields, but that's a topic for a future post. Here we're concerned with calculating the effect on a current of a magnetic field.

The essence of the argument is to apply the Lorentz force law to a collection of moving charges. In its most general form, if we have a charge distribution $\rho(\mathbf{r})$ with a velocity field $\mathbf{v}(\mathbf{r})$ immersed in a magnetic field $\mathbf{B}(\mathbf{r})$ then the total force on the charge due to the magnetic field is

$$\mathbf{F}=\int\mathbf{v}\times\mathbf{B}\rho d^{3}\mathbf{r} \tag{1}$$

In this general case, all factors in the integrand depend on position. This formula is often written as

$$\mathbf{F}=\int\mathbf{J}\times\mathbf{B}d^{3}\mathbf{r} \tag{2}$$

where $\mathbf{J}\equiv\rho\mathbf{v}$ is the volume current density, and represents the charge per unit area per unit time flowing past a given point.

We can make various simplifying assumptions to get some common special cases. A surface current is a current that flows over the surface of some volume, so the analogous formula is

$$\begin{aligned}\mathbf{F} &= \int\mathbf{v}\times\mathbf{B}\sigma da && (3)\\ &= \int\mathbf{K}\times\mathbf{B}da && (4)\end{aligned}$$

In this case, σ is the surface charge density and $\mathbf{K}\equiv\sigma\mathbf{v}$ is the charge per unit width per unit time flowing past a given point on the surface.

Finally, for a linear current (such as that in an infinitesimally thin wire), we get

$$\mathbf{F} = \int \mathbf{v} \times \mathbf{B} \lambda dl \quad (5)$$

$$= \int \mathbf{I} \times \mathbf{B} dl \quad (6)$$

Here, λ is the linear charge density and $\mathbf{I} \equiv \lambda \mathbf{v}$ is simply the charge per unit time flowing past a point in the wire, and is the current commonly used in describing electric circuits. Technically, $\mathbf{I}$ is a vector, since it has a direction, but in most circuit calculations, the directional aspect of current is ignored, since it's constrained to flow wherever the circuit takes it.

As an example of calculating the magnetic force on a linear current, suppose we have a square loop of wire of side length a lying in the yz plane with its centre at the origin. It carries a constant current I, flowing counterclockwise when viewed down the x axis (that is, looking towards negative x).

If we apply a constant magnetic field $\mathbf{B} = B\hat{\mathbf{x}}$, then the net force on the loop is zero. This follows since the force on the edge $y = \frac{a}{2}$ is

$$\mathbf{F}_{y=a/2} = a\left(I\hat{\mathbf{z}}\right) \times \left(B\hat{\mathbf{x}}\right) \quad (7)$$

$$= aIB\hat{\mathbf{y}} \quad (8)$$

That is, the right-hand side of the square experiences a force pointing to the right, since the current flows upwards and the magnetic field is pointing towards you (assuming you're standing on the positive x axis looking towards the origin). By a similar calculation, the force on the left-hand side of the square is $-aIB\hat{\mathbf{y}}$, since the current has reversed direction and the magnetic field is the same. Similarly, the top and bottom edges cancel out, giving a net force of zero.

Suppose we now impose a varying magnetic field of the form

$$\mathbf{B} = kz\hat{\mathbf{x}} \quad (9)$$

Notice that the field reverses direction as we cross the xy plane. To find the force, we can start with the top and bottom edges $z = \pm\frac{a}{2}$, since the field is constant along each of these edges. On the top edge $z = \frac{a}{2}$, $\mathbf{B} = \frac{ka}{2}\hat{\mathbf{x}}$ and $\mathbf{I} = -I\hat{\mathbf{y}}$ so

$$\mathbf{F}_{z=a/2} = a\left(-I\hat{\mathbf{y}}\times\frac{ka}{2}\hat{\mathbf{x}}\right) \tag{10}$$

$$= \frac{ka^2I}{2}\hat{\mathbf{z}} \tag{11}$$

On the bottom edge, $z=-\frac{a}{2}$, $\mathbf{B}=-\frac{ka}{2}\hat{\mathbf{x}}$ and $\mathbf{I}=I\hat{\mathbf{y}}$ so

$$\mathbf{F}_{z=-a/2} = \frac{ka^2I}{2}\hat{\mathbf{z}} \tag{12}$$

On the left and right sides, the forces cancel, since for each value of z there is a segment of the loop with current $+I\hat{\mathbf{z}}$ on the right and a corresponding segment with current $-I\hat{\mathbf{z}}$ on the left. The magnetic field is the same for these two segments so the net force is zero.

The total force is then

$$\mathbf{F} = \mathbf{F}_{z=a/2}+\mathbf{F}_{z=-a/2} \tag{13}$$

$$= ka^2I\hat{\mathbf{z}} \tag{14}$$

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